INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M.STAT FIRST YEAR
2014-16 SEMESTER-II

OPTIMIZATION TECHNIQUES

Final Examination

Answer any seven questions

Total Marks : 70 Duration : 3hours

(1) A transport company has two types of trucks, Type A and Type B. Type A has a refrigerated capacity of 20 m^3 and a non-refrigerated capacity of 40 m^3 while Type B has the same overall volume with equal sections for refrigerated and non-refrigerated stock. A grocer needs to hire trucks for the transport of 3,000 m^3 of refrigerated stock and 4,000 m^3 of non-refrigerated stock. The cost per kilometer of a Type A is \$30, and \$40 for Type B. How many trucks of each type should the grocer rent to achieve the minimum total cost?

(2) Consider the problem of minimizing $c^t x$ subject to $Ax \geq b$, $x \geq 0$, where $c \in R^n$, $A \in R^{m \times n}$ and $b \in R^m$. Let x_0 and y_0 be feasible solutions of the primal and the dual problems, respectively. Then show that a necessary and sufficient condition for x_0 and y_0 to be optimal solutions for the corresponding problems, is

$$y_0^t(Ax_0 - b) = 0, \text{ and}$$

$$x_0^t(c - A^t y_0) = 0$$

(3) Suppose A is an $m \times n$ matrix. Then show that exactly one of the following systems has a solution.

$$\text{System 1} : Ax < 0 \text{ for some } x \in R^n$$

$$\text{System 2} : A^t y = 0, y \geq 0 \text{ for some non-zero } y \in R^m$$

(4) Consider the following LP:

$$\begin{aligned} \text{P: Minimize} \quad & X_1 + X_3 \\ \text{Subject to } & x_1 + 2x_2 \leq 5, \\ & x_2 + 2x_3 = 6, \\ & x_1 + x_2 \, , x_3 \geq 0. \end{aligned}$$

(a) Write down the dual D of P and find the optimal solution of D graphically.
(b) Using the optimal solution of D, find the optimal solution of P.

(5) Find $x = (x_1, \ldots, x_n)^T$ to maximize

$$\frac{c^T x + \alpha}{d^T x + \beta}$$

subject to the constraints

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \text{ for } i = 1, \ldots, k$$

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \text{ for } i = k+1, \ldots, m$$

(6) Consider the balances transportation problem (BTP)with $c_{ij} > 0$ for all i and j. Let

$$a(\delta) = (a_1, \ldots, a_{i-1}, a_i + \delta, a_{i+1}, \ldots, a_m)$$
$$b(\delta) = (b_1, \ldots, b_{j-1}, b_j + \delta, b_{j+1}, \ldots, b_n)$$

Also, let β be the optimum basic set of variables for problem (BTP) with $a = a(0)$, $b = b(0)$ and let u and v be the corresponding simplex multipliers. When a is changed to $a(\delta)$ and b to $b(\delta)$, suppose that a basic set β remian feasible to (BTP) for all $0 \leq \delta \leq \delta_1$ for some $\delta_1 > 0$. Prove that the minimum objective value $f(a(\delta), b(\delta))$ is linear with slope $u_i + v_j$ in the interval $0 \leq \delta \leq \delta_1$ and that it is strictly monotonically decreasing in the interval $0 \leq \delta \leq \delta_1$ if and only if $u_i + v_j < 0$.

(7) Find the optimal solution for the following transportion problem

	D_1	D_2	D_3	D_4	*Capacity*
o_1	1	2	3	4	6
o_2	4	3	2	0	8
o_3	0	2	2	1	10
Demand	4	6	8	6	24

(8) The captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsmen at these position are as follows:

	Batting Position				
Batsmen	*I*	*II*	*III*	*IV*	*V*
P	40	40	35	25	50
Q	42	30	16	25	27
R	50	48	40	60	50
S	20	19	20	18	25
T	58	60	59	55	53

(a) Find a suitable assignment of batsmen to positions.

(b) If another batsmen U with the following average in batting position

Batting Position:	*I*	*II*	*III*	*IV*	*V*
Average runs	45	52	38	50	49

is added to the team, should he be included in the team. If so, who will be replaced by him.

(9) (a) Use the Big M-Method to find a optimal solution

$$\text{Min } Z = -3x_1 + x_2$$
$$x_1 - 2x_2 \geq 2$$
$$-x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

or

(b) Consider the following LP

$$\text{Max } \quad Z = 10x_1 + x_2$$
$$\text{Subject to } x_1 \leq 1$$
$$20x_1 + x_2 \leq 100$$
$$x_1, x_2 \geq 0$$

(i) Find all basic feasible solution for this LP.

(ii) Show that when the Simplex Method is used to solve LP, every basic feasible solutions must be examined before optimal solution is found.