INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M.STAT First Year
2014-15 Semester II

Time Series Analysis
Final Examination

Total Points 100. 27 April 2015 Duration: 3 hours

Note: • Notations and symbols as used in the class are followed. • Answer as many questions as possible, but the maximum you can score from these questions is 100. • Points are indicated at the end of each question.

1. Assume that $\{Y_t\}$ is an $ARMA(p,q)$ process with $ACF\ \gamma_Y$. Let $\{X_t\}$ be stationary with $EX_t = 0$ and with $ACF\ \gamma_X$. If $\gamma_Y = \gamma_X$, then prove that $\{X_t\}$ is also an $ARMA(p,q)$ process. Hint: You may use the fact that if $\{V_t\}$ is stationary with $E(V_t) = 0$ and if $\gamma_V(h) = 0$, for all $|h| > q$, then $\{V_t\}$ is an $MA(q)$ process. [20]

2. (a) The sunspot numbers $\{X_t, t = 1, \ldots, 100\}$ have sample autocovariances $\hat{\gamma}(0) = 1382.2$, $\hat{\gamma}(1) = 1114.4$, $\hat{\gamma}(2) = 591.7$ and $\hat{\gamma}(3) = 96.2$. Use these values to find the *Yule-Walker estimates* of ϕ_1, ϕ_2 and σ^2 in the model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + Z_t, \qquad Z_t \sim \mathrm{WN}(0, \sigma^2),$$

   for the mean corrected process $Y_t = X_t - 46.93, t = 1, \ldots, 100$. [5]

   (b) From the information given in the previous part, use *Durbin-Levinson algorithm* to compute the sample partial autocorrelations $\hat{\phi}_{11}, \hat{\phi}_{22}$ and $\hat{\phi}_{33}$ of the sunspot series. [5]

3. Let $P_n X_{n+h}$ denote the best linear predictor of values of the time series upto time n. Clearly it has the form, $P_n X_{n+h} = a_0 + a_1 X_n + \ldots + a_n X_1$.

   Show that the following two equations,

$$E[(X_{n+h} - a_0 - \sum_{i=1}^{n} a_i X_{n+1-i})] = 0$$

$$E[(X_{n+h} - a_0 - \sum_{i=1}^{n} a_i X_{n+1-i}) X_{n+1-j}] = 0, j = 1, \ldots, n$$

   determine $P_n X_{n+h}$ uniquely. [20]

4. In the *Innovations algorithm*, show that for each $n \geq 2$, the innovation $X_n - \hat{X}_n$, is uncorrelated with $X_1, \ldots, X_n$. Conclude that $X_n - \hat{X}_n$ is uncorrelated with the innovations $X_1 - \hat{X}_1, \ldots, X_{n-1} - \hat{X}_{n-1}$. [20]

5. Let a quality characteristic X be described by a discrete-time stationary Gaussian process X_t. Under this assumption, the process has a constant mean $E(X_t) = \mu$ and autocovariances are given by $\gamma(h) = Cov(X_t, X_{t+h}) = E(X_t - \mu)(X_{t+h} - \mu)$ where $\gamma(h), h = 0, \pm 1, \ldots$ is a function of lag h only. Under the stationarity assumption,

the process variance is a constant, $\sigma_X^2 = \gamma(0)$. Assume that X_t is a first-order stationary causal autoregressive Gaussian process with parameter ϕ, $X_t = \phi X_{t-1} + Z_t$ where $Z_t \sim N(0, \sigma_Z^2)$. In this case we have,

$$\sigma_X^2 = \frac{\sigma_Z^2}{1-\phi^2}.$$

Let us consider now the case of random measurement errors. We assume that the measurement error can be described by a Gaussian white noise process with constant variance $V_t \sim N(0, \sigma_V^2)$. Furthermore, X_t and V_t are assumed to be additively linked and stochastically independent. Thus the true process $\{X_t\}$ is NOT observed. Rather, the observable process is $\{X_t^e\}$. The observable quality characteristic is thus a Gaussian process $\{X_t^e, t = 1, 2, \ldots\}$ given by

$$X_t^e = X_t + V_t.$$

When $\{X_t\}$ is a first-order stationary $AR(1)$ process, $X_t - \mu = \phi(X_{t-1} - \mu) + Z_t$ then,

$$X_t^e = \phi X_{t-1} + Z_t + V_t.$$

Then variance of X_t^e becomes,

$$(\sigma_X^e)^2 = \frac{\sigma_Z^2}{1-\phi^2} + \sigma_V^2.$$

Find the MLEs of σ_Z^2 and σ_V^2 and ϕ. [15]

6. Consider the $SARIMA$ model, $X_t = Z_t + \Theta Z_{t-2}$. Identify the model using the notation $ARIMA(p,d,q) \times (P,D,Q)_s$. [5]

7. For the unemployment data, *(unemp)* of 'astsa' package of **R**, fit an appropriate $ARIMA/SARIMA$ model. Justify your choice. Use the fitted model to forecast for the next 12 months. [20]

8. Verify with short justifications or arguments whether the following statements are true:

   (a) *Innovation algorithm* is applicable only to the stationary time series.

   (b) AR and MA orders P and Q of $ARIMA(p,d,q) \times (P,D,Q)_s$ are used to fit $ARMA(P,Q)$ to the seasonal points of the given time series $\{X_t\}$.

   (c) Let $\{X_t\}$ be a stationary process and if $\rho(4) = 0.3$ and $\rho(h) = 0$ for all $h \geq 5$ then $\{X_t\}$ is an $MA(1)$ process.

   (d) *Durbin-Levinson algorithm* is used to estimate coefficients of AR models and *Innovations algorithm* is used to estimate parameters of $MA/ARMA$ models.

   (e) A better way of handling a long memory process is to consider fractionally integrated $ARMA$ or $ARFIMA$ models rather than to difference the series until stationary.

[2 + 2 + 2 + 2 + 2 = 10]

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