

INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M.STAT First Year
2014-15 Semester I

Non-Parametric and Sequential Analysis
Final Examination

Total Points 50.

Duration: 3 hours

Instructions

1. Definitions has no partial credit
2. Problems will get full credit only when the final answer is given
3. Hand written class notes are allowed in the examination hall

Attempt any number of questions

1. Suppose that the observations; 0.05, 0.04, 0.03, 0.49, 0.50, 0.66, 0.59, 1.17, 0.72, 0.83, are the samples generated from a continuous random variable.

- a) Find the 0.25 – th and 0.5 – th quantiles.
- b) Obtain a $100(1 - \alpha)\%$ confidence interval for 0.75 – th quantile using any method. [7]

2. Let (X_1, \dots, X_n) and (Y_1, \dots, Y_n) be random samples from F and G , respectively. Denote $X_{(1)}$ and $Y_{(1)}$ as the minimum order statistic based on n samples.

- a) Find $P(X_{(1)} > Y_{(1)})$.
- b) What will be the value of the probability obtained in (a) when $F = G$? [6]

3. Let (X_1, \dots, X_n) be a random sample from the uniform distribution on the interval (a, b) , where a and b are real unknown quantities. Let $X_{(j)}$ be the j th order statistic. Show that $Z_i = (X_{(i)} - X_{(1)}) / (X_{(n)} - X_{(1)})$, $i = 2, \dots, (n - 1)$, are independent of $(X_{(1)}, X_{(n)})$ for any a and b . [6]

4. Consider a continuous non-negative random variable X with distribution function F . Define a parameter

$$\Delta = 2 \int_0^\infty (1 - F(x)) dx - 2 \int_0^\infty (1 - F(x))^2 dx.$$

- a) Find an estimator of Δ based on U-statistics.
 - b) Find the asymptotic distribution of U-statistics obtained in (a) when X has exponential distribution with parameter λ . [7]
5. Obtain a plug in estimator $\hat{\Delta}$ of Δ which is defined in question number 4. Also discuss the bootstrap procedure to find the distribution of $\hat{\Delta}$. [6]
6. Derive the distribution of Chi-square statistic when the number of category associated with the Chi-square goodness of test is 2. [4]

7. Define the kernel function. Determine the order of the following kernel functions.

a) $k_1(x) = \frac{3}{4}(1 - u^2)I(|u| \leq 1)$.

b) $k_2(x) = \frac{2}{\sqrt{2\pi}}(3 - u^2)\exp(-\frac{u^2}{2})$. [5]

8. Let X_1, X_2, \dots be independently and identically distributed Bernoulli (θ), random variables. Discuss SPRT for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta = \theta_1 (> \theta_0)$. Also find the OC and ASN functions associated with this testing problem. [8]

9. Discuss Stein's two stage procedure for finding a fixed length confidence interval. Let (X_1, \dots, X_n) be random samples from normal distribution with mean μ and variance σ^2 . Verify whether the fixed length confidence interval for μ has specified confidence coefficient. [7]