INDIAN STATISTICAL INSTITUTE

CHENNAI CENTRE

M.STAT First Year

2014-15 Semester I

Large Sample Statistical Methods
Final Examination

Points for each question is in brackets. Total Points 100.
Students are allowed to bring 2 pages (front and back) of hand-written notes
Duration: 3 hours

1. (10) Let X_n be $AN(\mu, \sigma^2/n)$ and let Y_n be $AN(c, v/n), c \neq 0$, and put $Z_n = \sqrt{n}(X_n - \mu)/Y_n$. Show that Z_n is $AN(0, \sigma^2/c^2)$.

2. (10) Let $X_1, \cdots, X_n$ be a sequence of iid random variables from a distribution with finite first four moments. Show that the joint distribution of the first two sample moments is asymptotically normal using the Cramer Wold device.

3. (20) Suppose $X_1, \cdots, X_n$ are iid $U(\theta - 1/2, \theta + 1/2)$. Consider the one sample Wilcoxon statistic given by

$$W = \frac{1}{\binom{n}{2}} \sum_{i<j} I(X_i + X_j > 0)$$

for testing the hypothesis $\theta = 0$. Obtain the asymptotic distribution of W under the null hypothesis using the theory of U-statistics.

4. (15) Let $X_1, \cdots, X_n$ be iid Poisson observations with rate λ. Consider a Gamma(α, β) prior density for λ. Show that the posterior is consistent.

5. (10) Show that the likelihood ratio test is consistent.

6. (20) Show that the projection of Kendall's tau on the ranks is Spearmen's rho upto a constant. Derive the asymptotic distribution of Spearmen's rho under the null hypothesis of independence.

7. (20) Let $X_1, \cdots, X_n$ be iid according to the logistic distribution with cdf

$$F_\theta(x) = \frac{1}{1 + e^{-(x-\theta)}}$$

   (a) Show that the likelihood equation has unique root $\hat{\theta}_n$ that maximizes the likelihood function.

   (b) Find the asymptotic distribution of $\hat{\theta}_n$.

   (c) Show that $\overline{X}_n$ is a consistent estimator of θ.

   (d) Suggest an estimator that can be computed explicitly and has the same asymptotic distribution as $\hat{\theta}_n$.