Indian Statistical Institute
Chennai Centre
M.Stat. S.Stream I Year 2014-2015
First Semester
Semestral Examination

Sample Surveys and Design of Experiments 17.12.14

Answer as much as you can. The maximum you can score is 120.
The notation used have their usual meaning unless stated otherwise.
Time :- 3 hours

1. Consider a block design with v treatments and b blocks, each of size k.

   (a) Write down an appropriate linear model in the following form.

   $\underline{Y} = X\underline{\gamma} + \underline{\varepsilon}$, where X is a partitioned matrix of the form

   $$X = [\ X_0 \quad X_1 \quad \cdots \quad X_m\ ]$$

   for some m and $\underline{\gamma}$ is a vector, partitioned into a few vectors. Describe the X_i matrices, the components of $\underline{\gamma}$ and $\underline{\varepsilon}$.

   (b) What is an incidence matrix ? Express it in terms of the X_i's with justification.

   (c) Suppose the reduced normal equation for the treatment effects is $C\hat{\tau} = Q$. Find the expected value of $Q'\hat{\tau}$. Hence justify the use of it as the "sum of square for the treatments".

   (d) Consider the following block design.

Block 1 :	1	1	2	2
Block 2 :	3	3	4	6
Block 3 :	2	5	5	
Block 4 :	4	5		

   List all estimable treatment contrasts with justification.

   [6 + (2 + 4) + (10 + 3) + 5 = 30]

2. (a) When is a block design said to be connected ?

   (b) Consider a connected block design with its incidence matrix (N) satisfying

   $$n_{ij} = r_i k_j / n.$$

   (i) Show that the reduced normal equations for the treatment effects and the block effects are as follows.

   $$\begin{aligned}(D_r - (1/n)\underline{r}\underline{r}')\hat{\tau} &= \underline{T} - (1/n)\underline{r}G,\\ (D_k - (1/n)\underline{k}\underline{k}')\hat{\beta} &= \underline{B} - (1/n)\underline{k}G.\end{aligned}$$

   (ii) Show that the expressions for the treatment sum of squares and the block sum of squares are as given below.

   $$\begin{aligned}SS_{tr} &= \sum_{i=1}^{v} T_i^2 - G^2/n,\\ SS_{bl} &= \sum_{j=1}^{b} B_j^2 - G^2/n.\end{aligned}$$

   [2 + (6 + 8) = 16]

3. Consider a block design with v treatments and b blocks, each of size k. Suppose the block effects are i.i.d random variables with expected value 0 and variance σ_b^2 and the probability distribution of $\underline{\varepsilon}$ is as usual.

(a) Obtain $\Sigma = Cov(Y)$ and its inverse.

(b) Define error space and estimation space. Obtain them in terms of the X_i matrices.

(c) If $l'\tau$ is estimable, show how to find its BLUE.

(d) Let SS_E denote the error sum of squares under the usual fixed effects model. Show that the expected value of SS_E under the present model is $e\sigma^2$. Obtain the constant e in terms of the rank of the matrix X of Q1 (a).

[(3 + 5) + (2 + 2 + 3 + 5) + 7 + 7 = 34]

4. (a) Define a balance incomplete block design (BIBD).

(b) Suppose s is a prime or a prime power. Construct a BIBD with s^2 treatments and s^2+s blocks of size s each with justification. Illustrate with $s = 3$. Hence or otherwise construct an SBIBD with $(v = 13, k = 4, \lambda = 1)$.

[3 + (8 + 3 + 3) = 17]

5. The percentage of hardwood concentration in raw pulp and the cooking time of pulp are being investigated for their effects on the strength of the paper.

Suppose two concentrations (C) and two cooking times (T) are used. Explain what is meant by (i) the main effects of C and T and (ii) the interaction effect CT and how one can determine them.

[9]

6. (a) Suppose s is a prime or a prime power. Construct an $(s + 1) \times s^2$ orthogonal array of strength 2 with s symbols. Illustrate with $s = 3$.

(b) Describe how the array you constructed in Q(a) can be used to construct a fractional factorial design for a 3^4 experiment on nine blocks of size nine each. Determine the effect(s) confounded with the block effects. Justify each of your statements.

[(8 + 3) + (6 + 4) = 21]