INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE

M. STAT - I YEAR

END SEMESTER EXAMINATION (NOV. 12, 2014)

Time: $2\frac{1}{2}$ Hours MARKOV CHAINS **Max. Marks: 25**

(I) Consider a sequence of tosses of a coin with probability of getting heads p with $0 < p < 1$. At time n, the state of the process is the number of heads minus the number of tails in n tosses.

(i) Write down the transition probability matrix, P.

(ii) Is P irreducible? Why?

(iii) Is P bistochastic? Give reason. [2 + 2 + 1]

(II) Indian tax payers are classified into three types: (1) Honest (2) Some what honest (3) Dishonest- who do not pay any tax. It is found from the past (auditing of the income tax returns) 95% of type (1) continue to be the same type, 4% move to type (2), and 1% to type (3). It is found the figures are 6%, 90% and 4% for type (2), and 0%, 10% and 90% for type (3) tax payers.

(i) Express the problem as a Markov chain.

(ii) Find the stationary distribution of the Markov chain.

(iii) Statistics from the past data shows that type (2) tax payers do not pay taxes for Rs. 20, 000/- while type (3) do not pay taxes for Rs. 30, 000/-. Assuming an average income tax rate of 10% and assuming there are one crore Indians supposed to file income tax returns, determine the expected annual loss in the tax collection. [1 + 2 + 2]

(III) Let $X(t)$ be a pure continuous time Markov chain. Assume that

$$P(\text{an event happens in } (t, t+h)|X(t) = \text{even}) = \lambda_1 h + o(h),$$

and

$$P(\text{an event happens in } (t, t+h)|X(t) = \text{odd}) = \lambda_2 h + o(h),$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$. Assume $X(0) = 0$. Let $P_1(t) = P(X(t) = \text{odd})$ and $P_2(t) = P(X(t) = \text{even})$.

(a) Show that $P_1(t)$ and $P_2(t)$ satisfy the following differential equation: $P_1'(t) = \lambda_1 P_2(t) - \lambda_2 P_1(t)$ and $P_2'(t) = \lambda_2 P_1(t) - \lambda_1 P_2(t)$.

(b) Derive an explicit expression for $P_1(t)$ and $P_2(t)$. [3 + 2]

(IV) Examine whether the following statements are TRUE or FALSE. Give reasons in either case.

(i) Let P denote the transition probability matrix of order k. Here, k stands for the number of states. Assume k a finite positive integer. Then all k states cannot be transient.

(ii) Consider the following discounted dynamic programming problem with $S = \{1, 2\}$, $A = \{a_1, a_2\}$, and $\beta = 1/2$. Immediate rewards and transition probabilities are given below:

$$\begin{array}{ccc} & \text{State 1} & \text{State 2} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{bmatrix} 3/(1,0) \\ 6/(0,1) \end{bmatrix} & \begin{bmatrix} 2/(0,1) \\ 1/(1,0) \end{bmatrix} \end{array}$$

Here $3/(1, 0)$ means immediate reward is 3 and $q(1/f(1), a_1) = 1$ etc. Let $f(1) = a_1, f(2) = a_2$. Then f is not an optimal stationary policy for the discounted dynamic programming problem. [4 + 4]

Neatness and Brevity [2]