M.Stat I Year

Statistical Inference

End-Term Examination

Answer any 5 questions

Total Marks : 60 Duration : 3 hours

(1) Let $\theta = (0.\infty)$, Let R be the real line and let $L(\lambda, a) = (\lambda - a)^2$. Let the distribution of X be position with parameter $\lambda > 0.(\lambda \in \theta)$. Let $X_1, X_2, \ldots, X_n$ be i.i.d sample from poisson (λ). Let the prior distribution of λ be

$$g(\lambda) = (\tau(\alpha)\beta^\alpha)^{-1} e^{-\lambda/\beta}\lambda^{\alpha-1}, \text{ for } \lambda > 0$$

where $\alpha > 0$ and $\beta > 0$.

(a) Find the posterior distribution of λ.
(b) Find the Bayes rule with respect to the squared error loss function.
(c) Show that the usual maximum likelihood estimate is not a Bayes rule.
(d) Show that the M.L.E is a limit of Bayes rule.
(e) Show that the rule obtained in (iii) is a generalized Bayes rule with respect to the mean (improper prior)

$$\tau(\lambda) = \log \lambda(\tau(\lambda) = 1/\lambda \, d\lambda)$$

(2+2+3+3+3)

(2) In problem 1, consider a Bayesian test of $H_0 : \lambda \leq \lambda_0$ Versus $H_1 : \lambda > \lambda_0$

(a) Find the acceptance/rejection region of H_0
(b) If $\alpha = 5/2, \beta = 2$, the prior distribution is a $\mathcal{X}^2$-distribution with 5 degrees of freedom. Use $\mathcal{X}^2$-distribution (table) to perform the Bayes test.
(c) Let X have a discrete distribution with probability mass function

$$f(x/\theta) = \begin{cases} \theta & : f\, x = -1 \\ (1-\theta)^2\theta^x & if x = 0, 1, 2, \ldots \end{cases}$$

where $0 < \theta < 1$ Show that X is boundedly complete sufficient statistic but X is not a complete sufficient statistics for θ. (4+4+4=12)

(3) Show that $\bar{X}$ is admissible for estimating the mean of a normal distribution under squared error loss function. (12)

(4) Suppose that $X_i, i = 1, 2,$ are i.i.d Bermoulli(p)

(a) Show that the variance of MLE of p attains the Cramer- Rao Lower bound
(b) For $n \geq 4$, show that the product $X_1X_2X_3X_4$ is an unbiased estimate of P^4 and use this fact to find the UMV unbaised estimate of P^4 (12)

(5) (a) Let T be the unique Bayes estimate with respect to the prior density π. The show that the T is admissible.

(b) If an equalize rule is admissible, show that it is minimize (a decision rule 'd' is an equalizer decisaion rule, if $R(\theta, d)$ is the same for every value of θ) (6+6=12)

(6) A sample of five observations is taken on a $b(1, \theta)$ random variable to test. $H_0 : \theta = 1/4$ against $H_1 : \theta = 3/4$

(a) Find the most powerful test of size $\alpha = 0.05$.

(b) If $L(1/4, 1/4) = L(3/4, 3/4) = 0$, $L(1/4, 3/4) = 1$, $L(3/4, 1/4) = 2$ find the minimax rule.

(c) If the prior probability of $\theta = 1/4$ and $\pi_0 = 1/3$ and $\pi_1 = 2/3$ respectively, find the Bayes rule. (4+4+4=12)