Linear Models

End Semester Examination, November 2015

Answer as much as you can. Maximum you can score 40.

Q1. $y^{n\times1} = X^{n\times k}\beta^{k\times1} + e^{n\times1}$; $E(e) = 0, Var(e) = \sigma^2 I$. $rank(X) < k < n$. $l'\beta$ is estimable parametric function. Show that least square estimator of $l'\beta$ is unique. (5)

Q2. $y^{n\times1} = X^{n\times k}\beta^{k\times1} + e^{n\times1}$; $E(e) = 0$, $Var(e) = \Sigma$. $rank(X) < k < n$. $A = X(X'\Sigma^{-1}X)^{-}X'\Sigma^{-1}$. Show that A is same for any choice of $(X'\Sigma^{-1}X)^{-}$, (g-inverse). (5)

Q3. Consider the model $E(y_1) = 2\beta_1 - \beta_2 - \beta_3$, $E(y_2) = \beta_2 - \beta_4$ and $E(y) = \beta_2 + \beta_3 - 2\beta_4$ with usual assumptions. Describe the estimable functions. (5)

Q4. Find the minimum norm solution of the system of equations:

$$\begin{aligned} 2x + y - z &= 1 \\ x - 2y + z &= -2 \\ x + 3y - 2z &= 3 \end{aligned}$$

(5)

Q5. Consider the model $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_{11} x_{i1}^2 + \beta_{22} x_{i2}^2 + \beta_{12} x_{i1} x_{i2} + e_i$. where predictor variables take on the following values.

i	1	2	3	4	5	6	7
x_{i1}	1	1	-1	-1	0	0	0
x_{i2}	1	-1	1	-1	0	0	0

Show that $\beta_0, \beta_1, \beta_2, \beta_{11} + \beta_{22}, \beta_{12}$ are estimable and find (non matrix) algebraic forms for the estimates of these parameters. Find the MSE ($\frac{SSE}{d.f.}$) and the standard errors of the estimates. (10)

Q6. Consider the model $y_1 = \theta_1 + \theta_2 + \epsilon_1$, $y_2 = 2\theta_1 + \epsilon_2$ and $y_3 = \theta_1 - \theta_2 + \epsilon_3$ where $\epsilon_i, i = 1, 2, 3$ are $i.i.d. N(0, \sigma^2)$. Derive F Statistic to test $\theta_1 = \theta_2$. How do you test $\theta_1 = \theta_2 = 5$? (7+3)

Q7. Consider the model, $y_1 = \alpha + \beta x_1 + e_1$, $y_2 = \alpha + \beta x_2 + e_2$, $y_3 = \theta + \beta x_3 + e_3$, $y_4 = \alpha + \gamma x_4 + e_4$, $y_5 = \theta + \gamma x_5 + e_5$, $y_6 = \theta + \gamma x_6 + e_6$. Can you test $\alpha = \theta$ and $\beta = \gamma$ simultaneously? If yes derive test Statistic, if no justify. Answer the same when $x_3 = x_1 + x_2$ and $x_4 = x_5 + x_6$, x_i's are all known. (7+3)

Q8. For any matrix X, show $X(X'X)^{-}X' = XX^{+}$, X^{+} is Moore Penrose inverse. (5)

Q9. A idempotent implies $rank(A) = trace(A)$. Is converse true? If yes then prove it, if no then give a counter example. (5)

All THE BEST :)