INDIAN STATISTICAL INSTITUTE

CHENNAI CENTRE

M.STAT FIRST YEAR 2015-16 SEMESTER-I

STATISTICAL INFERENCE

Mid Semester Examination

Total Marks : 30 Duration : 2-1/2hours

Answer any Six Questions

(1) Suppose X_1 *and* X_2 are iid observations from the pdf

$$f_\theta(x) = \theta x^{\theta-1}\exp^{(-x^\theta)}, x > 0, \theta > 0.$$

Show that $logX_1/logX_2$ is an ancillary statistic.

(2) Let X be a discrete random variable with

$$P_\theta(X = x) = \frac{\binom{\theta}{x}\binom{N-\theta}{n-x}}{\binom{N}{n}}\, x = 0, 1, 2, \ldots,\ \min\{\theta, n\},\ n - x \leq N - \theta$$

where n and N are positive integers, $N \geq n$, and $\theta = 0, 1, \ldots N$. Show that X is complete.

(3) Let $(X_1, \ldots, X_n)$ be a random sample from $f_\theta(x) = \exp^{-(x-\theta)} I_{(\theta,\infty)(x)}$, where $-\infty < \theta < x < \infty$, is an unknown parameter.

(a) Use basu's theorem to conclude $X_{(1)}$ and S^2 are independent.

(4) Let $(X_1, \ldots, X_n)$ be a random sample from a distribution on $\mathcal{R}$ having the Lebesgue density $\theta^{-1}\exp^{-(x-\theta)/\theta} I_{(\theta,\infty)(x)}$, where $\theta > 0$ is an unknown parameter.

(a) Find a statistic that is minimal sufficient for θ.

(b) Show whether the minimal sufficient statistics in (a) is complete.

(5) Let $(X_1, \ldots, X_n)$, $n > 2$, be a random sample from the uniform distribution on the interval $\theta_1 - \theta_2, \theta_1 + \theta_2$, where $\theta_1 \in \mathcal{R}$ and $\theta_2 > 0$. Find the UMVUE's of θ_j, $j = 1, 2$, and θ_1/θ_2.

(6) For each of the following distributions, Let $(X_1, \ldots, X_n)$ be a random sample. Is there a function of θ, say $g(\theta)$, for which there exists an unbiased estimator whose variance attains Cramer- Rao lower bound.? If so, find the same.

(a)

$$f_\theta(x) = \begin{cases} \theta x^{\theta-1} & 0 < x < 1, \theta > 0 \\ 0 & \text{otherwise,} \end{cases}$$

(b)

$$f_\theta(x) = \begin{cases} log(\theta).\theta^x/(\theta-1) & 0 < x < 1, \theta > 1 \\ 0 & \text{otherwise,} \end{cases}$$

(7) Let $(X_1, \ldots, X_n)$ $n > 2$, be a random sample from the exponential distribution on (a, ∞) with scale parameter θ. Show that $\sum_{i=1}^n (X_i - X_{(1)})$ and $X_{(1)}$ are independent for any (a, θ), where $X_{(j)}$ is the jth order statistics;

(8) Suppose that $(X_1, \ldots, X_n)$ are iid with a $beta(\mu, 1)$ and $(Y_1, \ldots, Y_n)$ are iid with a $beta(\theta, 1)$ pdf. Also X s are independent of Y s.

(a) Find the likelihood ratio test of

$$H_0 : \theta = \mu \text{ versus } H_1 : \theta \neq \mu.$$

(b) Show that part a) can be based on the statistics

$$T = \sum_{i=1}^n logX_i \ / \ (\sum_{i=1}^n logX_i + \sum_{i=1}^n logY_i)$$