INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M. Stat. (NB Stream) – Semester I
2015 – 2017
Linear Algebra and Linear Models
Mid-term Examination (Linear Algebra)

Total Marks: 50 . Maximum Marks: 45

Date: 22 August 2015 **Duration:** 2 hours

1. Prove or disprove the following.
   (a) Let S be a subspace of V. Let $A \subseteq S$ and $B \subseteq V \setminus S$ be linearly independent. Then $A \cup B$ is linearly independent. [3]
   (b) Let R, S and T be subspaces of V. If $R + S \subseteq S + T$ then $R \subseteq T$. [3]
   (c) Let S and T be subspaces of V. Let $\mathbf{x}$ and $\mathbf{y}$ be vectors in V. If $\mathbf{x} + S \subseteq \mathbf{y} + T$ then $S = T$. [3]
   (d) If A is an $m \times n$ matrix such that $m > n$ then $A\mathbf{x} = \mathbf{0}$ has a non-trivial solution. [3]
   (e) Let A be an $m \times n$ matrix. If $AX = \mathbf{0}$ implies $X = \mathbf{0}$, then the matrix A has a left inverse. [3]
   (f) Let A be an $n \times n$ matrix. If $\rho(A) = \rho(A^2)$ then $\mathcal{C}(A) \cap \mathcal{N}(A) = \{\mathbf{0}\}$. [4]
   (g) Let R, S and T be subspaces of V. If $R \cap S = S \cap T = T \cap R = \{\mathbf{0}\}$ then the sum $R + S + T$ is direct. [3]

2. Define Hermite canonical form (HCF). Show that a matrix in HCF is idempotent. [6]

3. Let A and B be matrices with the same number of columns. Show that $A\mathbf{x} = \mathbf{0}$ and $\begin{bmatrix} A \\ B \end{bmatrix} \mathbf{x} = \mathbf{0}$ have the same solution space iff $\mathcal{R}(B) \subseteq \mathcal{R}(A)$. [4]

4. Define g-inverse of an $m \times n$ matrix A. Show that an $n \times m$ matrix G is a g-inverse of A iff $\rho(I - GA) = n - \rho(A)$. [6]

5. The following statements about an $n \times n$ matrix A are equivalent.
   (i) A is a projector,
   (ii) A is idempotent,
   (iii) $\mathcal{N}(A) = \mathcal{C}(I - A)$,
   (iv) $\rho(A) + \rho(I - A) = n$,
   (v) $\mathcal{C}(A) + \mathcal{C}(I - A)$ is direct.

   [10]

6. Find the matrix of the identity transformation of $\mathbb{R}^3$ with respect to the ordered bases $\mathcal{X} = \{(1,1,1)^T, (1,0,1)^T, (1,1,0)^T\}$ and $\mathcal{Y} = \{(0,1,0)^T, (0,0,1)^T, (1,0,0)^T\}$. [2]