INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE

M.STAT. (NB - stream) - YEAR I

2015 – 2016

ANALYSIS I

Time: 3 Hours MID-SEMESTER EXAMINATION **Marks: 40**

Answer any 8 questions. All questions carry equal marks.

1. Let A^B denote the set of all functions from the set $\cancel{A}$ B to the set $\cancel{B}$ A. Let A, B, C be 3 non-empty sets. Are the sets $C^{A \cup B}$ and $C^A \times C^B$ bijective ? Justify your answer.

2. Let $\alpha, \beta \subseteq \mathbb{Q}$ be cuts. Will there be any problem if we define the multiplication of cuts as follows: $\alpha \cdot \beta = \{x \leq r.s | x \in \mathbb{Q}, r \in \alpha, s \in \beta\}$? Also, if we take an arbitrary intersection of cuts, will it form a cut ? Justify your answers.

3. Let (X, d) be a metric space. Define $d' : X \times X \to \mathbb{R}$ by $d'(x, y) = min\{d(x, y), 1\}$. Does d' form a metric on X ? Justify your answer.

4. Let (X, d) be a metric space, and A and B disjoint closed subsets of X. Show that there exist disjoint open sets G_1 and G_2 such that $A \subset G_1$ and $B \subset G_2$.

5. Let (X, d) be a metric space, $x \in X$, and $r > 0$. Let $E = \{y \in X | d(x, y) \leq r\}$. Show that E is a closed set. Is E necessarily the closure of $N_r(x)$? Justify your answer.

6. Let X, Y be metric spaces, and $f : X \to Y$. Give a proof or provide a counterexample: If f is continuous then inverse images of compact sets under f are compact.

7. Let X, Y, Z be metric spaces, $f : X \to Y$, $g : Y \to Z$. Let $h = g \circ f$. Suppose g is continuous, bijective, and Y is compact. Show that f is uniformly continuous if h is so.

8. Let X, Y be metric spaces, and $f, g : X \to Y$ be continuous functions. Suppose that f and g agree on $E \subset X$. Do they agree on $\overline{E}$ as well ? Justify your answer.

9. Let $a_1, a_2, a_3, \ldots$ be positive numbers. Suppose $\sum a_n$ converges. What can be said about the convergence of $\sum \sqrt{a_n a_{n+1}}$? Justify your answer.

10. Let $a_n = 1/n^2 + 1/(n+1)^2 + \ldots + 1/(2n)^2$. Does $\lim\limits_{n \to \infty} a_n$ exist ? Justify your answer.