INDIAN STATISTICAL INSTITUTE

Chennai Centre

M.STAT I year 2014-15 Semester II

Time Series Analysis

Mid-Semester examination

Maximum Marks 50

Date: 26 February 2015 Duration: 3 hours

Note: ♣ Notations and symbols as used in the class are followed. ♣ Answer as many questions as possible, but the maximum you can score from these questions is 50. ♣ Marks are indicated at the end of each question.

1. Let $\{W_t\}$ and $\{Z_t\} \sim i.i.d$ random variables with $P(W_t = 0) = P(W_t = 1) = 1/2$ and $P(Z_t = -1) = P(Z_t = 1) = 1/2$. Define

$$X_t = W_t(1 - W_{t-1})Z_t.$$

Show that $\{X_t\}$ is White Noise but not $i.i.d.$ [5]

2. For a Moving Average process of order 1, MA(1), that is, $X_t = Z_t + \theta Z_{t-1}$ where $Z_t \sim WN(0, \sigma^2)$ show that the autocovariance function is

$$\gamma(h) = \begin{cases} (1+\theta^2)\sigma^2 & h = 0 \\ \theta\sigma^2 & h = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

and the autocorrelation function is

$$\rho(h) = \begin{cases} \frac{\theta}{1+\theta^2} & h = \pm 1 \\ 0 & h > 1 \end{cases}$$

Can we say that MA(1) models are always unique? Obtain conditions under which the MA(1) model is invertible. $[3 + 3 + 1 + 3 = 10]$

3. Let $\{Y_t\}$ be a stationary time series with mean 0 and covariance γ_Y. If $\sum_{j=-\infty}^{\infty} |\psi_j| < \infty$ then prove that the time series

$$X_t = \sum_{j=-\infty}^{\infty} \psi_j Y_{t-j} = \psi(B) Y_t$$

is stationary with mean 0 and autocovariance function

$$\gamma_X(h) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \psi_j \psi_k \gamma_Y(h+k-j)$$

[5 + 5 = 10]

4. Autoregressive process of order 1, AR(1), is defined as a stationary solution $\{X_t\}$ to the equations $X_t = \phi X_{t-1} + Z_t$ where $Z_t \sim \mathrm{WN}(0, \sigma^2)$, $|\phi| < 1$ and Z_t is uncorrelated with X_s for each $s < t$. Using the results from the previous problem, (evn if you have not attempted it, you can use it),
   (a) compute a stationary solution to the AR(1) process and
   (b) also show that the solution obtained in the first part above is unique. [5 + 5 = 10]

5. Determine the order of the following ARMA(p, q) processes:
   (a) $X_t + 1.9X_{t-1} + 0.88X_{t-2} = Z_t + 0.2Z_{t-1} + 0.7Z_{t-2}$
   (b) $X_t - \frac{9}{4}X_{t-1} - \frac{9}{4}X_{t-2} = Z_t - 3Z_{t-1} + \frac{1}{9}Z_{t-2} - \frac{1}{3}Z_{t-3}$

   Are these models causal and invertible? [5 + 5 = 10]

6. Verify with small justification whether the following statements are true (T) or false (F):
   (a) Every MA(q) process is a q-correlated process. [T / F]
   (b) Every stationary q-correlated process is MA(q) process. [T / F]
   (c) Every weakly stationary process is either a linear process or can be transformed into a linear process by subtracting a deterministic component. [T / F]
   (d) A sequence of independent Cauchy random variables is weakly stationary. [T / F]
   (e) If $X_t = Z_t + \theta Z_{t-1}$ where $\{Z_t\} \sim$ iid $(0,\sigma^2)$ and θ's are real constants then $\{X_t\}$ is strictly stationary. [T / F]

   [2 + 2 + 2 + 2 + 2 = 10]

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