Non-parametric and Sequential Analysis

Midterm Examination **02 March, 2015**

Total Mark: 60 **Time: 2hours**

Instructions

1. Hand written class notes are allowed in the Examination hall
2. Answer any number of questions and each one caries equal Mark
3. Only best six will be counted for grading.

1. Define empirical distribution function. Show that empirical distribution is a sufficient statistics. Find the covariance between empirical distribution functions at different points x and y where $x < y$.

2. Define a quantile function. Consider a continuous random variable X with distribution function F. Obtain a $100(1-\alpha)\%$ confidence interval for the quantile function.

3. Let $(X_1, ..., X_n)$ be a random sample from the uniform distribution on the interval $(\theta_1 - \theta_2, \theta_1 + \theta_2)$, where θ_1 and θ_2 are real unknown quantity. Find the UMVUEs of θ_j, $j = 1, 2$. (Hint: Use Lehmann-Scheff theorem)

4. Let $(X_1, ..., X_n)$ be a random sample from the uniform distribution on the interval $(\theta - 1/2, \theta + 1/2)$, where θ is real unknown quantity. Let $X_{(j)}$ be the jth order statistic. Show that $(X_{(1)} + X_{(n)})/2$ is strongly consistent for θ.

5. Consider a continuous random variable X with distribution function F. Define a parameter

$$\Delta = Cov(X, F(X)).$$

Find an estimator of Δ based on U-statistics.

6. Write a bootstrap procedure to find the distribution of the estimator obtained in question number 5.

7. Find a plug in estimator of Δ defined in question number 5. Write down the procedure for finding the bias due to jackknife.

8. Find the distribution of Chi-square statistic when the number of category in the Chi-square goodness of test is 2.