Financial Time Series Analysis

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The models for analyzing financial time series, may be divided in to two classes, namely, observation driven and parameter driven. In observation driven models, the volatility is assumed to be a function of the past observations as in the cases of ARCH and GARCH models. On the other hand, in parameter driven case, the conditional variances are generated by some latent models. That means we need models for generating conditional variances as a functions of some unobserved factors. The Autoregressive sequences of non-negative rvs may be used for this purpose. In this talk the main focus is the different aspects of parameter driven models, known as Stochastic Volatility Models (SVM). The log-normal SVM by Taylor (see Taylor (1994)) is the simplest and the best known example to describe the evolution of the return series, Y_t:

$$Y_t = \epsilon_t e^{h_t/2}, \quad h_t = \mu + \phi(h_{t-1} - \mu) + \sigma_\eta \eta_t, |\phi| < 1, t = 1, 2, ...$$

where h_t represents the log-volatility, which is unobserved but can be estimated using the observations, ε_t and η_t are two independent Gaussian white noises, with unit variances. Due to the Gaussianity of η_t, this model is called a log-normal SV model. A major task here is to estimate $\theta = (\mu, \phi, \sigma_\eta^2)$. The likelihood function of θ based on $y = (y_1, y_2, ..., y_T)$ is the pdf of y given θ. But, from the model structure, it is clear that the likelihood function will depend on the unknown vector $\mathbf{h} = (h_1, h_2, ..., h_T)$. Hence we may get an explicite form of the likelihood function by integrating out the latent variables. In view of this, we may write the likelihood function as

$$\begin{aligned} L(\theta) &= f(y|\theta) \\ &= \int f(y, \mathbf{h}|\theta) d\mathbf{h} \\ &= \int f(y|\mathbf{h}) f(\mathbf{h}|\theta) d\mathbf{h}. \end{aligned}$$

The marginal likelihood over the parameters of the SVM is defined by a T-dimensional intergral of the above type. The multiple integral $L(\theta)$ cannot be factored in to a product of T one-dimensional integrals because of the dependence of h_t on the past.

Due to the difficulties in obtaining explicit forms of MLEs, several numerical methods are proposed in the literature. Markov Chain Monte Carlo (MCMC) is a commonly used method for numerical estimation. Several algorithms for this method are also available in the literature.

The development of SVM requires a stationary stochastic model to generate volatlities. In lognormal SVM, a Gaussian AR(1) model is used to generate log-volatilities and then exponentiated it to get a sequence of Markov dependent log-normal sequences. We would like to introduce some non-Gaussian models for generating volatilities, and propose a class of product autoregressive models. In particular, the details of gamma and Weibull SVM will be discussed. While introducing Weibull SV models, we need to discuss the properties of an AR model to generate sequences of Gumbel extreme value (GEV) rvs. Detailed analysis of SVM induced by the GEVAR(1) model and the related inference problems along with simulation and real data analysis will be presented.

Some Refernces

Tsay (2005). *Analysis of Financial Time Series.* 2^{nd} *edition.* Wiley Interscience.
Taylor, S. J. (1994). Modelling stochastic volatility. *M*athematical Finance 4, 183-204.