AGM Belief Revision about logic

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- Background and Motivation
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Contents

Background and Motivation

- *On the Logic of Theory Change* (AGM 1985)

- Three kinds of operations:

Expansion	Contraction	Revision
Add a belief	Remove a belief	Consistently add a belief
$K + p$	$K - p$	$K * p$

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  - (1) Remove beliefs that are incompatible with “Biden will win”
    - $K - \neg b$
  - (2) Add “Biden will win”
    - $(K - \neg b) + b$

Background and Motivation

- Belief about logic:
  - *Modus ponens* is valid
  - For all ϕ, ϕ implies ϕ
  - Every statement is true or false.
  - No statement is both true and false.
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- Revision examples: Graham Priest, J. C. Beall, students…

Background and Motivation

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  - What do we contract?
  - How do we expand?

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- We need to revise by the logical principles under which belief sets are closed.

Groundwork

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(4) There is minimal loss of non-logical information through the revision (Maximality)

Groundwork

- Logics are Tarskian consequence operators:

  Given some language $\mathcal{L}$ and sets $X, Y \subseteq \mathcal{L}$,

  (1) $X \subseteq Cn_L(X)$

  (2) $X \subseteq Y \Rightarrow Cn_L(X) \subseteq Cn_L(Y)$

  (3) $Cn_L(X) = Cn_L\big(Cn_L(X)\big)$

- Belief sets are sets of formulas K^L closed under some logic L (i.e. $K^L = Cn_L(K^L)$)

Groundwork

- What is “non-logical information?”
  - ϕ is “non-logical” relative to a belief set K^L iff ϕ is contained in some belief base for K^L.
  - $\mathbb{B}^{K^L} = \bigcup\{B : B \text{ is a belief base for } K^L\}$ is the set of all non-logical information for K^L.

Groundwork

Postulate Name	Postulate in symbols	Postulate in English
Closure	$K^L * L' = Cn_{L'}(K^L * L')$	An agent's new collection of beliefs is closed under the new logic.
Coherency	$K^L * L'$ is trivial iff L' is the trivial logic	An agent's new collection of beliefs is trivial iff the new logic is the trivial logic.
Origin	There is some $A \subseteq \mathbb{B}^{K^L}$ for which $K^L * L' \subseteq Cn_{L'}(A)$	All an agent's new beliefs are either old non-logical beliefs or consequences of these and the new logic.
Maximality	For all $\phi \in \mathbb{B}^{K^L}$, if L' is not the trivial logic, then $\phi \notin K^L * L'$ iff $Cn_{L'}\big((K^L * L') \cup \{\phi\}\big)$ is trivial	The only non-logical beliefs not preserved through revision are those that must be omitted on pain of triviality.

Results

- Remainder Set
  - The set of all maximal, non-trivial closures (under the new logic) of non-logical information.

$K^L \perp L'$ is the set of all $Cn_{L'}(X)$ such that
(1) $X \subseteq \mathbb{B}^{K^L}$ (2) $Cn_{L'}(X) \neq \mathcal{L}$ (3) For all Y such that $X \subset Y \subseteq \mathbb{B}^{K^L}$, $Cn_{L'}(Y) = \mathcal{L}$ Or $\{\mathcal{L}\}$ if no such X exists

Where $\mathbb{B}^{K^L}$ is the union of all belief bases for K^L

Results

- Preference Orderings

$\leq$ is a total, well-founded, preorder on $\wp(\mathcal{L})$

$\leq$ is semi-strict iff for any $X, Y \subseteq \mathcal{L}$ if $X \nsubseteq Y$ and $Y \nsubseteq X$, then $X < Y$ or $Y < X$

Results

Basic revision operator:

$$K^L * L' = \bigcap min_{\leq}(K^L \perp L')$$

$\leq$ can be any preference ordering

Strong revision operator:

$$K^L *_s L' = \bigcap min_{\leq}(K^L \perp L')$$

$\leq$ is a semi-strict preference ordering

Results

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1. Gather the non-logical information $\mathbb{B}^{K^L}$
2. Identify all the candidate new belief sets
3. Pick the "best" ones among these
4. Take what is common to the best ones

Results

Example: $K = Cn_{LP}(\{p, \neg p\})$ and the new logic is classical logic

1. $\mathbb{B}^{K^L} = \{p, \neg p, \neg\neg p, \neg p \rightarrow p, \neg p \land \neg p, \dots\}$
2. Two elements of the remainder set: $Cn_{CL}(\{p\})$and $Cn_{CL}(\{\neg p\})$
3. We pick our favorite(s) using $\leq$
4. Take the intersection of our favorites

Results

- Theorem 1: Basic revision satisfies closure, coherency, and origin, but not maximality

Yes!	Closure	$K^L * L' = Cn_{L'}(K^L * L')$
Yes!	Coherency	$K^L * L'$ is trivial iff L' is the trivial logic
Yes!	Origin	There is some $A \subseteq \mathbb{B}^{K^L}$ for which $K^L * L' \subseteq Cn_{L'}(A)$
No….	Maximality	For all $\phi \in \mathbb{B}^{K^L}$, if L' is not the trivial logic, then $\phi \notin K^L * L'$ iff $Cn_{L'}\big((K^L * L') \cup \{\phi\}\big)$ is trivial

Results

- Theorem 2: Strong revision is characterized by the postulates closure, coherency, origin, and maximality.

Yes!	Closure	$K^L * L' = Cn_{L'}(K^L * L')$
Yes!	Coherency	$K^L * L'$ is trivial iff L' is the trivial logic
Yes!	Origin	There is some $A \subseteq \mathbb{B}^{K^L}$ for which $K^L * L' \subseteq Cn_{L'}(A)$
Yes!	Maximality	For all $\phi \in \mathbb{B}^{K^L}$, if L' is not the trivial logic, then $\phi \notin K^L * L'$ iff $Cn_{L'}\left((K^L * L') \cup \{\phi\}\right)$ is trivial

Some Relevant Work

- Alchourron CE, Gardenfors P, Makinson D. (1985) "On the Logic of Theory Change: Partial Meet Contraction and Revision Functions." *JSL*
- Delgrande JP, Peppas P, Woltran S. (2018) "General Belief Revision." *JACM*
- Mares ED. (2002) "A Paraconsistent Theory of Belief Revision." *Erkenntnis*
- Ribeiro MM. (2013) *Belief revision in non-classical logics*. Springer, London ; New York
- Tanaka K. (2005) "The AGM theory and inconsistent belief change." *Logique & Analyse*
- Fermé E and Hansson SO. (2018) *Belief change*. Springer Berlin Heidelberg, New York, NY