Homework 3

Logic for Computer Science

1. Show that a formula of modal logic is satisfiable iff it is satisfiable in an acyclic model (one where the frame is (W, R) such that for no $w \in W$, we have wR^*w).

2. Let (M, u) and (N, v) be model - point pairs, and E_1 and E_2 be bisimulations on them. Then is $E_1 \cap E_2$ also a bisimulation on them (and why / why not) ?

3. In each case below, consider modal logic defined over frames where the accessibility relation R satisfies the property as specified. In each case, give formulas valid over models based on these frames, and discuss whether the canonical model has the property given below.

   (a) for all $w, w_1, w_2 \in W$, if $(w, w_1) \in R$ and $(w, w_2) \in R$, then there exists $w_3 \in W$ such that $(w_1, w_3) \in R$ and $(w_2, w_3) \in R$.

   (b) R is reflexive, transitive and has no maximal elements.

4. Are the properties of irreflexivity and anti-symmetry definable in basic modal logic? Justify your answers.

5. Consider modal logic over equivalence frames. Give a complete axiom system for its valid formulas and a decision procedure for satisfiability.

6. Give an algorithm to check satisfiability of a given modal formula over reflexive and transitive models.

7. Consider modal logic with the accessibility relation $R = (W \times W) \setminus Id$. That is, the modality is a difference operator, interpreted over worlds different from the current world. Is this logic decidable? Is this operator already definable in basic modal logic? Give a complete axiom system for its valid formulas.