Lecture 1

08.01.2024

What is logic?

1. a set of rules on some grammar
2. to reason about statements

- **mathematical logic**

  study of mathematical reasoning

What is our language to do such a study?

A game of Y/N questions!

- **Guess a number**

1. Is the number even? Y
2. Is the number < 100? Y
3. Is the number < 50? N

⋮

- Guess a group!

1. Is it finite? Y

2. Is it cyclic? Y

3. Is the order of the group prime? N

⋮

- Guess a graph!

1. Is it a complete graph? Y

2. Is the graph connected? Y

3. Are the number of nodes ≤ 5? Y.

Natural numbers

$0, S, <, >, +, \cdot, |, =$

$a < b$:

$\exists x((a+x=b) \wedge \neg(x=0))$

Groups

$*$, e.

Commutativity

$\forall x \forall y (x * y = y * x)$

Graphs.

E: edge relation

$\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$: logical connectives

not, and, or, implies, iff

$\exists$, $\forall$: quantifiers.

$x_1, x_2, x_3, \ldots$: variables

Common symbols that we use to talk about mathematics.

- Language of natural numbers

$(0, S, +, \cdot, <)$ $(=)$

0: constant symbol; S, $+$, $\cdot$: function symbols; $<$: relation symbol

$\boxed{\begin{array}{c} S : \mathbb{N} \to \mathbb{N} \\ S(0) : 1 \\ S(S(0)) : 2 \end{array}}$

- Language of groups

$(e, *, \ )$ $(=)$

e: constant symbol; $*$: function symbol

- Language of graphs

$(\ , \ , E)$ → relation symbol → predicate symbols

First-order languages.

Parameters of the language

$(\mathcal{C}, \mathcal{F}, \mathcal{P})$

$\mathcal{C}$: a countable collection of constant symbols

$\mathcal{F}$: a countable collection of function symbols

$\mathcal{P}$ a countable collection of predicate symbols

- for each $f \in \mathcal{F}$, $\#(f)$ gives the arity of f.s. f.

- for each $p \in \mathcal{P}$, $\#(p)$ gives the arity of p.s. p

Examples : $2x+3y$, $5x_1 + 7x_2 + 9x_3 + 10x_4$.

$2x + 3y = 5x + 7y$ terms

$5x + 2y + 6z = 0$ formulas

Formulas :

- A primitive formula is formula
- If φ and ψ are formulas then, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \to \psi$, $\varphi \leftrightarrow \psi$, $\neg\varphi$, $\neg\psi$ are formulas.
- If φ is a formula and x is a variable then $\forall x\, \varphi$, $\exists x\, \varphi$ are formulas

What is a primitive formula?

We answer this in the next class.