Lecture 1

het's start with playing a grassing game It's a game with Yes No questions Guess a number ! 1. Is the number even? Y 2. In the number divisible by 3? N 3. Is the number < 1007 Y 4 Is the number < 50 7 N Guess a group ! 1. Lo it abelian? N 2. In the order of the group even ? Y Guess a graph ! 1. Is the graph planar ? N 2. Is the number of vertices < 10 7 y What togic is all about ? It is all about studying reasoning

in a formal on matthe matical way Now, we will focus on mathematical reasoning. Natural mumber $0, S, \chi, \gamma, t, \cdot, l, =$ $a < b : \exists n((a + x = b) \land \neg (n = 0))$ Groups Graph E: VnVy(nEy > yEn) Some symbols that are common : lo great convectives



First-order language Parameters of the language (& , F , &) l: a comtable collection of constant symbols F: a commtable collection of function symbols p: a commtable collection of relation symbols - for each f E J , # (f) denotes the arily of f - for each p E p , # (p) denotes the arity of p #j: J→ IN Z Any function on predicate symbol #j: P→ IN J is associated with an arity. In. a way, we have the alphabet of om first-order language, the common symbols above, and the parameters. Now, we are set to describe the words and sentences .

Formulas



What is a primitive for mula 2





Term - Any veriable is a term. - Any constant symbol is a term. If t, t2, ---, tn are linns, and f is an n-any function symbol, then f t, t2 -- th is a term

Prime twe formulas - If ti, tr are terms, then ti= tr is a primitive formula. - If t, tr, ..., t, are time, and p is an n.any relation symbol, then, ptitz-...tu is a primitive formula Examples (On members) - x+y term n y ; term - n.y < y.z: formular In ty (a ky): bormula $-(x=x)\wedge\gamma(x=x)$; formula First-order logic : Syntan Alphabet l = {c, c, --... }: a commtable collection of

constant symbols

. a conntable collection of function symbols $- \vec{f} = \left\{ \vec{f}_1, \vec{f}_2, \cdots \right\}$ $- \left\{ \begin{array}{ccc} & & & & \\ & - & \\ \end{array} \right\} \left\{ \begin{array}{ccc} & & & \\ & & \\ \end{array} \right\} \left\{ \begin{array}{ccc} & & & \\ & & \\ \end{array} \right\} \left\{ \begin{array}{ccc} & & & \\ & & \\ \end{array} \right\} \left\{ \begin{array}{ccc} & & & \\ & & \\ \end{array} \right\} \left\{ \begin{array}{ccc} & & & \\ & & \\ \end{array} \right\} \left\{ \begin{array}{ccc} & & & \\ & & \\ \end{array} \right\} \left\{ \begin{array}{ccc} & & \\ \end{array} \right\} \left\{ \left\{ \left\{ \begin{array}{ccc} & &$ · a commtable collection of relation symbols. $- \bigvee - \left\{ \chi_{1}, \chi_{2}, \right\}$ a comtable collection of variables · = equality symbol \neg , V, A, \rightarrow , $\langle \rightarrow \rangle$ connectivos. - Y, <u>]</u> · quantifiers -), (bracherts Jums (J). $t := \pi i | c_i | f_i t_i t_2 - t_i , c_i \in C, \pi i \in V$ Formulas (F) $\varphi, \gamma := (t_1 = t_2) | \phi_i^{i_1} t_1 t_2 \cdots t_{i_j} | (-1(\varphi)) | (\varphi \vee \varphi) |$ $(q \land \psi) \mid (q \rightarrow \psi) \mid (q \leftrightarrow \psi) \mid (\forall \uparrow (q)) \mid (\exists \uparrow (q))$ $t_k \in \mathcal{J}$, fraultk (Backus Nam form)