Lecture 10

bet I be a set of formulas, and q be a formula in CPL. We define ME q (semantic consequence relation) and T+q (deductive consequence relation as contin . Soundness the one m If I't q then I't q We have already discussed the proof procedure for soundness theorem. So, once we show that the anioms are valid and the rules preserve consequence, we are done But, we do not know what these anions and rules are - We now focus on the proof of completimens theorem





 $3. \psi (M.P. 1, 2)$ This completes the proof of the if-part. (only-if part) but TU & F V & To prove that I' + q -> y - ble prove this by applying induction on the length of a divination / proof of y from rU{q] pare lase: (n=1): Then y is a men ber of MU {cp} or y is an anion Case L' Y = P Then, we have to show MFq -> q [Let un consider an aniom : q -> q] We have MFQ->Q. Cure 2: VPEP Then, we have to show MI-q-, Y. Since YET, we have THIY

[Let us consider an anion: Y - (q - y)] Then, we have the following derivation of Q, y from M. M H I. Y (Premise) 2. y → (q → y) (Ariom) $3 \quad q \rightarrow \psi \quad (M \cdot P \cdot 1, 2)$ Case 3. y is an anion To show MI + Q -> W. We have the same derivation as above. M H 1 4 (Arison) 2. $\gamma \rightarrow (\phi \rightarrow \gamma)$ (Anion) 3 Q->4 (M.P. 1,2)-So, we are done with the base cases To prove the base cases, we consider Anion (1) Q > Q (2) Q - (4 - Q) Rule e e y

I.H. Suppose the result holds for all derivations of length < m. I.S. Suppose the length of the derivation of y from MV 203 is m+1. Then, by the definition of F, Y is either on asion or, a member of TU{p] or, obtained by some application of some rule. We have already dealt with the first two possibilities. For the third possibility we assume that y is obtained by an appication of M.P. (Why? - Because M.P. is the only rule that we have considered till now) Since y is obtained by M.P., there exist y; and y in the direvation of y from MUEP we have $\Gamma \cup \{ \varphi \} \vdash \Psi i$ and $\Gamma \cup \{ \varphi \} \vdash \Psi i \rightarrow \Psi$. We have $\Gamma \cup \{ \varphi \} \vdash \Psi i$ and $\Gamma \cup \{ \varphi \} \vdash \Psi i \rightarrow \Psi$.





het us vous note that we can derive q- q from the following areans, using MP. $(') \quad \varphi \rightarrow (\psi \rightarrow \psi)$ $(2) (\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi))$ HW Find such a derivation of 9-19 So, we have the following collection of anions and rule till nors: Anioms $1. \quad \varrho \rightarrow (\psi \rightarrow \varrho)$ $2. \quad (\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varrho \rightarrow \psi) \rightarrow (\varrho \rightarrow \chi))$ $3(\neg \varphi \rightarrow \psi) \rightarrow (\neg \varphi \rightarrow \psi) \rightarrow \varphi)$ Rule: q q-) 4 (M.P.)