

Complete asignatization of FOL

FOL Syntan



show that the anions are valid and the miles preserve con sequence. Then, we get som dues by applying induction on the length of a proof of q from M. For completeness, we also proceed as before, that is we assume That I H of and show that MH q. What are the steps? Step L: We in the duce the notion of consistency A set of formulast is said to be inconsistent if there is a formula q st. MFq and MF7q. Min consistent if it is not inconsistent. Stelp 2: MH q iff TV {7 q} a consistent Stip 3 Every consistent set of formulas is satisfiable

We concentrate on proving Step 3.

How do we show Step 3 ? Slep 1 Extend a consistent set T to a consistent, complete and witness-fulfilled set Δ of formulas. Stelp 2: Défine a model Mrs based on D Slefe 3. Prove the truth lemma : for all formulas q, $M_{\Delta} \neq q$ iff $q \in \Delta$. Then, we would have that A is satisfiable and hence, T is . Ilms we would finish the completeness proof forward, we introduce the notions of substitution and substituta bility, which are integral to the proof of complete ness that we would deal with in the following .

Sules titution .

Jerms: Let t and t' be two terms and n be a variable be write t[t/n] for t' replacing a in the term t -n(t/n) = t' $- y[t/n] = y, y \neq n$ $-C \left(\frac{t}{n} \right) - C$ $- f_{i}^{R} t_{i} \cdots t_{k} [t_{k}] - f_{i}^{R} t_{i} [t_{k}] - \cdots t_{k} [t_{k}]$ Formulas i. Let q be a formula, t be a term, x be a variable be write q[t/n]for a replaced by t in q. $-(t_i = t_j) \begin{bmatrix} t_n \end{bmatrix} = t_i \begin{bmatrix} t_n \end{bmatrix} = t_j \begin{bmatrix} t_n \end{bmatrix}$ $- p_{i}^{k}(t_{i}\cdots t_{k}) \begin{bmatrix} t_{n} \\ t_{n} \end{bmatrix} - p_{i}^{k}t_{i} \begin{bmatrix} t_{n} \\ t_{n} \end{bmatrix} - t_{k} \begin{bmatrix} t_{n} \\ t_{n} \end{bmatrix}$ $-(1\psi)[t/n] = -(\psi[t/n])$ $-(Y \vee X)[t_n] = Y [t_n] \vee X[t_n]$



When should we say . ME Y[t/n]? $\mathcal{M} \models \Upsilon \begin{bmatrix} t/n \end{bmatrix} \stackrel{\cdot}{\to} \mathcal{M} \qquad \mathcal{M} \begin{bmatrix} n \to y(t) \end{bmatrix} \models \Psi, \quad \mathcal{M} = (D, \mathbb{I}, \mathcal{Y})$ Would this always hold ? φ : $\exists x (r(x = y))$ $\varphi[2/y]$: $\exists n(\tau(x=n))$ As we can see, q is satisfiable, but q ["/y] is not. So, the answer to the above question is NO When should it hold, or when can we substitute a variable with a term in a formula? Sub sti tu kabi kity A term t is said to be substitutable for a variable n in a formula & when I a formula & when the hold ! follo im g con ditions

- q is atomic tivel always be substitutable for n in q. - q is ry: t is substitutable for n in y. - q is yVX; t is sub for n in y and t is sub. for n in X - q in YAX'. 11 $-\varphi$ is $\psi \rightarrow \chi$: 1) - q 'n y <> X! າ - q in Yyy: x does not a cam free in f OR y doe not appear in the term t and t is substitutable for n in y. · yy: 11 This completes the definition of substitute bility of a variable by a term in a formula. What remains to be done is to prove the following proposition :

Proposition: Let q be a formula, n be a variable t be a timm, and M be a model. Suppose t us substitutable for n in q. Then, $M \models \varphi[t/n] \quad \mathcal{M} \quad \mathcal{M}[n \rightarrow y(t)] \models \varphi$ $\mathcal{M} = (D, 1, \mathcal{Y}).$ H.W. Prove this proposition det in now get back to the proof of Step 3, that is every comis tent set of FOL formula has a model, that is we need to construct a model (D, I, Y) p.t. $(D, I, Y) \models M$. Now,

from CPL we already know that . M ann be entended to a consistent and

complete set, A say, which is in fact a model set. We need to find a model Ms = (Ds, Is, Ys) n.t. Ms F Q

iff QED. In the next class we will try to find this Ms and explore what witness - fulfilled ness is in this regard