## Lecture 17

## SAT PROBLEM



## EXPRESSIVITY

FOL is way more expressive than CPL

## A LOGIC IN BETWEEN CPL and FOL



Dyntan ! Fix a countable set of atomic prop-ositions P = {p, p2, --- }.  $\varphi, \psi' = I | T | \varphi | 2 \varphi | \varphi \vee \psi | \varphi \wedge \psi | \varphi \rightarrow \psi |$ 9 <> 4 | [] 9 | \$9 One can consider formulas of the form Semantics : There are various ways to provide semantics to model formulas. \* Topological semantics. Neighborhood semantics \* Algebraic semantics Co-algebraic semantics ₽ Kripke semantics ¥

(Kripke) Frame ;

A frame is a fair F= (W, R), where W is a a non-empty set, and R C WXW

is a binary relation.

(Kripby) Model A model is a pain M = (F, N), when F = (W, V) is a frame and  $V: W \rightarrow 2^{P}$ is a valuation punction.

We refer to W as a set of (possible) world.

· When p E V (w), we say that p hold at no im M, otherwise p is false

at w im M.

Given a model M, and a world w in M, (M, w) is termed as a pointed model.

Truth definitions :

The notion 'q holds at the world w in the model M', denoted by M, 15FP is defined inductively as follows:  $- \mathcal{M}, \omega \models \neg q \quad \mathcal{M} \qquad \mathcal{M}, \omega \not\models q.$  $- \mathcal{M}, \omega \models \varphi \vee \psi \quad \mathcal{M} \quad \mathcal{M}, \omega \models \varphi \quad \alpha, \quad \mathcal{M}, \omega \models \psi$ - M, w F Q N Y M, W F Q and, M, w F Y -  $M, \omega \neq q \rightarrow \psi M M, \omega \neq q implie M, \omega \neq \psi$  $-M, w \models q \leftrightarrow \psi \psi M, w \models q \psi M, w \models \psi$  $-M, \omega \models \Box \varphi$  iff for all  $\omega' \in W$  with  $\omega R \omega'$ ,  $\mathcal{M}, \omega' \models \varphi$  $M, w \models Q q$  iff there exists w'  $\in W$  ranch that w R w' and  $M, w' \models q$ . & q is satifiable if there is a model M = (W, R, V) and some wEW, ot

 $\mathcal{M}, \omega \models \varphi$ .

\* q is valid iff its negertion is not patis fi able.





Note that □-formulas are always pat.
inplate in states is where {w': (w, w') ∈ R]

is empty.

Examples of valid formulas - D(PV7P): Valid.



Tom definatility to (in) distinguishability M: wi Is it possible to distingu ish wand was by PML w2 ish wand was by PML H.W. Show that (M, w2) and (M, w3) satisfy the same model formulas. Jill now we were considering worlds/states of the same model. Let us now consider the notion of distinguishing worlds of different models.  $\mathcal{N}$ M w2 P

Can you dis tinguish the states is, in M and &, in M, that is, can you dis tinguish between the pointed model (M, w,) and (N, v,) by a modal formula ? (Assume here that I is not in your language ) IS q distinguishes (M, w), and (N, v), as it holds at w, but not at v, DVA DIY would all work as a dis tin quis hung for mula What about (M, w, ) and (N, v) now ? U<sub>1</sub> P la, Core N': v<sub>2</sub> v<sub>3</sub> V M : w2 •

Actually, no modal for mules can distinguish between (M, w) and (N, v). How do we prove this ? Jo find a better way to prove such results, we now introduce the following concepts. Modal equivalence Jwo pointed model (M, w) and (N, v) are said to be modelly equivalent if for all model formules \$\mathcal{P}\$, \$M\$, \$\overline{P}\$ if N, U F Q, that is, they satisfy the same modal for nulas. Can we bring out a notion of invariance betroen the pointed model so as to capture the notion of moal equivalence given above 2