Lecture 19

FOL VS. PML

How minihan / different they are ?





FOL formula: = = y, = = y2 = y2 (2(y1= y2) A 7 (y2= y3) A 7 (y3=y1) A Ray, A Ray, A Ry, y3 A Ry, y3

This formula holds in M' but not in M So, modal logic cannot differentiate bet ween these two models but first.

order logie can. Ilms we can see that FOL is more expressive than PML. What is the precise relationship between FOL and PML? We first need to consider the language of FOL to deal with PML Parameters of the language. R: a binary relation symbol. Po, P1, P2, ---- : unary relation symbol Thus, a Kripke model M: (W, R, V) com be seen as an FOL-structure (D, F) as follows: D = W;  $\Gamma(\mathbb{R}) = \mathbb{R} \subseteq W \times W$ ;  $\widehat{\Gamma}(P_i) = V(P_i)$ Standard tranlation

A translation of PML formulas into

FOL formulas is given as follows: We con sider a variable or to provide this translation, lenown as the standard translation (STx)  $ST_n(k_i) = P_i x$  $ST_{n}(\bot) = \tau(zzz)$  $ST_{n}(q) = 7 ST_{n}(q)$  $ST_n(qVy) = ST_n(q) \vee ST_n(y)$  $ST_n(q\Lambda \Psi) = ST_n(q)\Lambda ST_n(\Psi)$  $ST_{1}(q \rightarrow \psi) = ST_{1}(q) \rightarrow ST_{1}(\psi)$  $ST_{2}(q \leftrightarrow \psi) = ST_{1}(q) \leftrightarrow ST_{2}(\psi)$  $ST_{n}(Q \varphi) = \exists \gamma (R_{ny} \land ST_{y}(\varphi))$ STr (□q) = Yy (Rry → STy(q)) Find STr  $(\Diamond (\square p) \rightarrow q))$ Answer: Iy (Rmy A (V2 (Ry2 ->P2) -> gy))

Model examples FO-structures het us consider the following as p d d d a 7 6 V c V M М Dy1 2 {a, b, c, d, e} DM = {1,2,3,4,5}  $L_{\mu'}(p) = \{a, d\}$  $I_{u}(P) = \{1, 3\}$  $T_{n}(g) = \{2, 4, 5\}$  $I_{M'}(Q) = \{b, c, n\}$  $I_{n}(R) = \{(1,2), (2,3),$  $f_{\mu'}(\mathbf{k}) = \{(a,b), (a,c)\}$ (b, d), (c, d), (d, e)(3,4), (3,5) Some related results L. For all Kripke models M and all worlds w in M: M, w E q iff M, FSTr (q) for all model for mulas of.



Cese 3: q := DY. Suppose M, wFDY. Then, there exists v in M such that who and M, v F Y. Since who, we have ! M [r + w, Z + v] F R r Z. Since M, v F Y, we have M[r + v] F STr(Y) (by I.H.). het us consider a new variable y, say. Then, we have M[y=v] = STy (y) and M[z=v, y=v] = Rny Now, as a does not occur free in  $ST_y(y)$ , we have  $M_{E^{2,y}}, y \rightarrow y \neq ST_y(y)$ . So, we have MENNW, y > v] F Rmy N STy(y) Then,  $M_{[2\rightarrow\omega]} \neq \exists y (R_{xy} \wedge ST_y(y))$ Thus MERNE FSTR (QY) -Conversely, suppose that MEZ-10] FST2(SY) So, MERNEJ F Zy (Ray A STy (Y)). Then,



An infinite set of - Compact ness theorem

modal formulas is satisfiable if every finite subset of it is satifiable. H.W. Prove this result using the standard translation. Lowenheim - Skolem theorem: If a set of model formulas is satisfiable in an infinite model then it is satisfiable in models of every infinite cardinality H.W. Prove this result using the standard trank lation. So any PML for mula can be expressible in the relevant FOL but conversely (Recall the models and the FOL formula which we stanted our discussion) Can we think of a collection of FOL formulas which are equivalent to the PML formulas 2