Lecture 20.



PML formula: (Dp -> q) $ST_{n} (\land (\square P \rightarrow q))$ = $\exists \gamma (R_{n\gamma} \land (\forall n (R_{\gamma}x \rightarrow P_{x}) \rightarrow g_{\gamma}))$

What about the converse? Can every first-order for mula in two variables be equivalent to the translation of a moder formula? Ans. NO - Consider Q := R22. Suppose 4 is a model formula such that STx(4) is equivalent to q. Consider the models: We have: Mge->wj = p and Nge->vj = p But (M, w) ~ (N, v), and so they satisfy the same model formulas. So, q cannot be equivalent to STr (4) for any modal for mula y. Hence the result. Can we characterize some set of first-

order formulas that are equivalent to the translation of model formulas 2 Ans. Yes Those for mulas that are invariant under bis imme lation. A first order for mula q(a) is roard to be invariant under bis impedation if for all Knipke models M and N, all worlds w in M and v in N and all bisi mulations Z between M and N and that is Zo, we have ! $\mathcal{M}_{[n \rightarrow \omega]} \neq \varphi(n)$ iff $\mathcal{N}_{[n \rightarrow \omega]} \neq \varphi(n)$ van Benthem Characterization theorem Let q(2) be a first order form la with one free variable n. Then, q(n) is invariant under bisi mulation eff q(a) is logically equivalent to a standard translation of a model formula. The proof of the if - part is trivial The proof of the only-if part is quite involved We do not discuss it here.

Finite model property (FMP) A logic λ is said to satisfy the finite model property if any satisfiable formula q in λ is satisfiable in a finite model Dow FOL has FMP? H.W. Provide an answer to the question above Does PML has FMP? Yes. In what follows we will prove this result In fact, we prove a stronger result. Strong finite model property (SFMP) A logie h is said to satisfy SFMP if ang satisfiable formula q in h is satisfiable in a finite model of at most 2191, where 191 denotes the number of sub formulas of q. Model logic (PML) satisfies Proposition SFMP

Irrof: Let q be a model formula. fit Sub (9) denote the set of subformulas of the formula q. Then, Sub(q) is a finite pet. Assume that q is satisfiable Then there is a model M = (W, R, V) and a world w in M such that, M, w F q. Jo get om result we have to reduce the size of M is such a way that the satisfiability of q is preserved somehow. A natural way to reduce size is to partition the domain set W, so we need to define some equivalence relation on W. While doing so, we have to consider the satisfiability of q in some way, in per ti cular the satisfiability of the formulas in Sul (q). Through all these, we hope to preserve the satisfiability of q

in the smaller model. het us define a relation ~q on W as follows: For all w, v E W, way viff for all formulas vin Sul (q), M, w F Y H M, v FY Is ~ q an equivalence relation ? Yes (Check!) Then, ~q partitions Wints equi valence classes pet [12] denote the equivalence class of w in W, and let WN = {EW3/WEW} How many elements do Wa have? het us define a map g: W_ -> & (Suble) g([w]) = { y e Sub (q) : M, w F 4 }

 $So, g([w]) \subseteq Sub(q)$. Is g well-defined? We have to show that : if way, then g(EwJ)=g(EwJ) Suppose war v. Then M. wEY'ny MUEY for all y in Sub (@) Hence, $\mathcal{J}([w]) = \mathcal{J}([v]).$ Lo g injective ? Let w, v E W, such that g([w]) = g([v]). Then, $\{\psi \in Sub(\varphi) : \mathcal{M}, w \neq \psi\}$ = $\{\psi \in Sub | q\}$: $\mathcal{M}, \forall \not\models \psi \}$. So, $\mathcal{M}, \upsilon \not\models \psi$ if M, v = y for all y e Sub (q). So, $\omega \sim v$, Athet is $[\omega] = [v]$. So, γ is an imjective map from Wn to 8 (Sulla) Since $|p'(Sub(q))| = 2^{|q|}$, we have ' $|W_{\sim}\rangle \leq 2^{|\varphi|}$ So, starting with any W, we get a finite

bounded set W~ . Now we need to define a binary relation R. on W. and a valuation V., such that the satisfaction of the formulas in Sub (9) do not get affected, that is, for all YE Sub(q), M, w = y iff M, [w] = q, ieith Mr = (Wr, Rr, Vr) · Let us first define V_{n} : $V_{n}(b) = \{ [w] : w \in V(b) \}, for$ all proposition al variables. How do we define R~ 2 Let is post pone this dis cursion and get to the proof of the following Lemma . For all for mules y E Sub (q). and for all w in M, M, w F 4 if $\mathcal{M}_{\mathcal{N}}$ [w] = $\mathcal{V}_{\mathcal{N}}$ Let's continue with the proof in the vent class.