Lecture 23

We will take care of Step 2 now. We construct a map inductively from . V of the q. - Lablean to W of the given model, h: V > W as follores Take h(vo) = wo Now, consider any v in V such that h(v)=10, ray. Define A(v) = M(v). Then, if \$ q  $\in \lambda(v), \quad \Diamond q \in \mu(w) \cdot S_0, \quad \mathcal{M}, w \neq \Diamond q \cdot$ So, there is some us such that wRw' and  $M, \omega' \neq \varphi$  So, we have,  $\{\forall \mid \Box \forall \in M(\omega)\}$  $V\{Q\} \subseteq M(w') \in ATQ$ . Then we have a v' with  $\lambda(v') = \mu(w') \supseteq \{\psi \mid \Box \psi \in \mu(w)\}$  $v \{\varphi\}$ . This in sures that we can continue Constructing the go-lab lean, that is, Step 2 is ensured. Thus, if Qo is satisfiable Then the algorithm terminates successfully. Conversely, suppose that the algorithm ter minules successfully. To show the qo is satisfiable. Let The tablean constructed

by the algo in then be  $(V, E, v_0, \lambda) = \overline{I}q_0$ Define a Kripbe model M: (V, E, V), where  $\overline{V}(p) = \{ v : p \in \lambda(v) \}$ . Now,  $\varphi_0 \in \lambda(v_0)$ . Let as prove the following claim. Claim: For all  $v \in V$ ,  $q \in SF(q_0)$ ,  $uf q \in \lambda(u)$ then M, v F q. Assuming the claim, we have  $M, v_0 \neq \varphi$  as  $\varphi_0 \in \lambda(v_0)$ . Let us now prove the claim. We prove by applying induction on the pize of q. Bese case :  $q := p : p \in \lambda(v) \Rightarrow v \in \widetilde{V}(p)$ ⇒ M, v FÞ.  $\varphi' = \tau \varphi : \tau \varphi \in \lambda(\upsilon) \Rightarrow \varphi \notin \lambda(\upsilon).$  $\Rightarrow \lor \notin \widetilde{V}(\flat).$ => M, v F 7 p Induction hypothesis : Suppose the result holds for all formulas of size < n. Induction step: Let q be a formula of size

n+1. We have the following cans  $: \forall \land \chi \in Y(v)$ Case L: q:= y VX  $=) \gamma \in \lambda(v) \cap \chi \in \lambda(v)$  $\Rightarrow$   $M, v \not\models \gamma$   $n \quad M, v \not\models \chi$ => M, 0 F Y VX Case 2:  $q := \gamma \Lambda X : \gamma \Lambda X \in \lambda(v)$ =) VEX(v) and XEX(v) =) M, v F y and M, v F X => M, v = YNX. q:= DY ! Let Dy E > (v). To show Case 3' that M, v F DY. Take any v, p.t. v E v'. To phone! M, v' F Y. Since v Ev' and  $\Box \forall \in \lambda(v), v \in \lambda(v)$ .  $So, M, v \neq \gamma$ .  $q' = \Diamond \psi'$ : Let  $\Diamond \psi \in \lambda(v)$ . To phow ! Case 4  $\mathcal{M}, v \not\models \Diamond \psi$ . Since  $\Diamond \psi \in \lambda(v)$ there is a with v E v and  $\Psi \in \mathcal{M}(\mathcal{V})$ , and hence,  $\mathcal{M}, \mathcal{V} \models \Psi$ So,  $M, v \neq Q \Psi$ .

This completes the proof of the claim as well as the proof of the correctness of the algorithm. A brief discussion on complexity. The size of the tree is exponential in the size of the given formula. So , the time complexity is quite high. However, if we consider space complexity, some clear use of space would show no that dproblem is in PSPACE. In fact, the patisfaction problem for model logic is PSPACE hand as well, and Thus PSPACE. complete . In the discussion on the decision procedure of the sotisfaction problem, we have considered all possible Kripke models. So we have the follo wing natural question.

How do we get such decision procedures with respect to a restricted class of models, eg., reflexive, symmetric, transitive me dels ? H.W. (optional) Find decision procedures for the above problems. Model-checking problem In a logie A, the model-checking problem co "itutes the following: Given a model M, and a formula P, check whether M satisfies q. Model-checking in model logic ! Given a model M and . a w in M, check whether M, w F Q. A similar pro plem! Given a model M and a formula &, find all the worlds in M that satisfy Q H.W. Give algonithms for both the model-checking problems in model logic. M.W. (optional) Give the run time of your algorithms in terms of sizes of the model and the formula