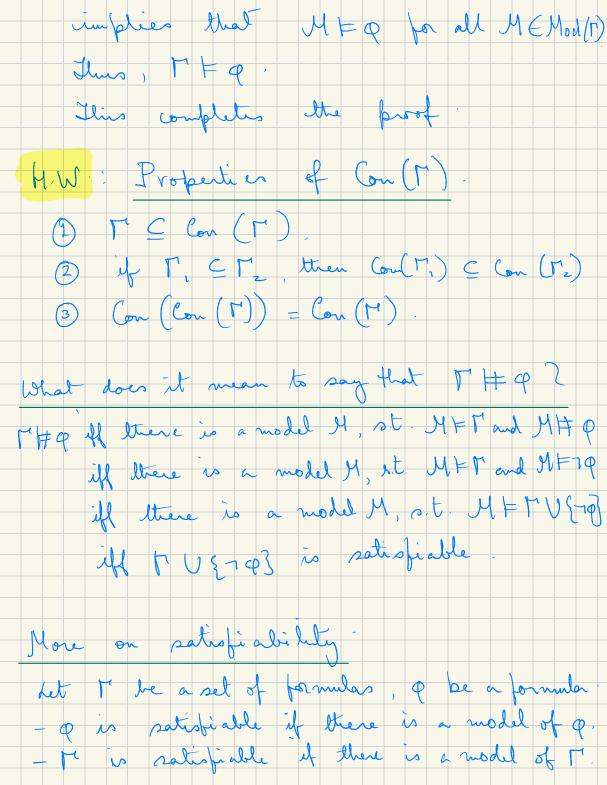
LECTURE 5 24.01-2024 The use about of the symbol F P = φ (entails) H F P (ratisfies) Models and Theories Given any formula P, define Mod (P) = EM/M=P3. Given a set of formulas V¹, define Mod (r°) = {MM≠r3 Let K be a class of models The theory of K, denoted by Th (K), is defined by Th (St) = { p | M + p for all models II in K ? Let I', I'z be two sets of formulas s.t.

Mod (M) = Mod (M2) Let Jd, K, be two classes of models n.t. Ki E Kz. Then, Th (&) = Th (\$\frac{1}{2}) Consequence of a set of formulas Let I' be a set of formulas The consequence of M, devoted by Con (r) is defined as (on (r) = Th (Mod (r)) Proposition bet I be a set of formulas
and q be a formulae We have: Q & Con (I) Proof: Suppose MFQ. To show: QE Can(r)
Now, all models in Mod (r) satisfy
Q. So, QE Th (Mod (r)), that is, QE Con (T). Conversely, suppose that $Q \in (on(T)$ To show: $\Gamma \vdash Q$ Noo $Q \in Th (Mod (\Gamma'))$



Examples - Consider an f. o. l L. with parameters - M = {p| 23 ! In M satisfiable? To answer in the affirmative, we have to find a model (D, I, y) s.t. (D, I, y) & Pn Then, (D, I, 14) F P, 2 , as y(2) & I(P) Then, $(0,1,1) \models P_x$, as $l_y(a) \in J(P_1)$. (3) D = (1) . I(P) = {n ∈ N | n is even }.

y' V -> D: lg(y) = 2 if y= n 0 y xx Thus, T is indeed satisfiable. - M = {P/n, 7 P/x}, un satisfiable Γ = { ρ'x , γ x , ¬ (p, x Λ p, x)}:

uuratistiable un sati stiable Sinilarly we can find unsatisfiable sets of familes of any find size k with k > 2 What about infinite sets of formulas? In the same way, if a set has any of the above finite collections of Johnna Now, suppose I is one infinite set of formulas s.t. all finite subsets of I are satisficule. What happens then?

Compact ness Theorem of F.O.L. Let M be om infinite set of formulas. Then I' is satisfiable iff It is finally satisfiable (every finite subset of Mis satisfiable) Non triveal part If I is fine try satisfiable (fin-sat), then I is satisfiable (sat) How does this result connect with the consequence relation? 1) If M is pin-sat then M is sat 2 If T = \phi , then there in finite subset Mo of Most. No FQ.

Result: 1 off 2 -Proof @ > 1) : Let I' be fin act. To show that I is sat Suppose not. Then
I'm formulas q. Then, there is a formula is, say of the first and M = 7 p So there are. - [Cfin [1.t.] + 7 } by (2)
- [2 fin [2] omd My is not satisfiable 1. a con tra diction. Ruce, the result (D=) (2) Let 1 = q To show that there is to Equ T' s.t. To F Q. Suppose not 'So, for all To Efin To # P So, for all Mo Star T, Mo V Eroz is sat Then by D MU { rq3 is sat. Then, THQ, a contradiction Hence the result

No re applications having arbitrarily large finite models.
Then I has an infinite models. Proof let D-Edi, dr, --- 3 be a countable collection of new constant symbols not occurring in the Consider $\Delta S = \Gamma \cup \{ \gamma(di = dj) | i, j \in IN \}, i \neq j \}$ Now, Mis satisfiable. So, M is finetely satisfiable. Take any finete subset of { 7 (di = dj) | i, j \in N, i \ j \} Such a finite set will be satisfiable in a model of thering that many distinct elements. So, we have that & is finitely satisfiable.

So, by compact news theorem I is not spiable. But, a model of D has infinitely many elements Now, MED. So, a model of Dis also a model of M. Thus I has on infinite model This completes the proof. Definability It, say, is said to be first-order definable of there is a set of serlences ri s.t. Mod (1) = K. H.W. Let FIN denote the class of all binite structures - Show that FIN is not first-order definable