Lecture 8

What abort the kind of mathematical reasoning that we do while trying to prove various results ? To study this for melly, we introduce the following notion: Déductive consequence relation M': a set of formulas q: a formula. M-q is a deductive consequence relation . Déductive consequence relation can be defined in various ways, e.g., Hilbert-style ani on atis ation, Gentyins sequent calculus, and Gentzen's natural

deduction '



of & (L) X L, where L denotes the langnage under consideration. De write an instance of such a rule as <u>Y1, Y2, ---, Yk</u>, <u>Y</u> where {Y1, Y2, --, YK} constitute the fremese of the rule, and of is the consequence of the rule. - An application of a rule basically means that when ri, ri, - , rk appear in some pequence, Q, Q2, - -, Qn, pay, P can be withen as a next step in the sequence How are the two con sequence relations related 2 Soundness Theorem: If M+q, then Mit q. flow to prove this result?

What kind of properties would we like to prove for these anions and mes to get our soundness them ? het us explore . Suppose I t Q Then we have: $\Gamma \vdash Q_1$ $\Gamma \vdash Q_2$, $\Gamma \vdash Q_2$ To prove $\Gamma \models \varphi_n (= \varphi)$ $\Gamma \vdash Qn (= Q)$ Here $q_1, q_2, \dots, q_n(=q)$ is a sequence used in the definition of $\Gamma + q$. The above possibilities suggest the fast that an approach to prove the someties result could be to use induction on the length of the sequence ve have considered in the definition of FFQ