Lecture 9



Induction Slep (n=m+1; m>1): q, can be (i) a member of M (ii) an axiom (iii) obtained by a application of some rule of inference. We have already seen how to tackle Cases (i) and (ii). Now we move on to Case (iii) \* To take care of Case (iii), we need to show that if any model satisfies the deremines of a rule, it would also catisfy the consequence of the rule In other words, we very that, reles preserve consequence relation. Thus to prove Soundness, we show' (i) Anions are validities. (ii) Rules preserve consequence The proof then follows.

Proving Doundness (if MEQ, then MEQ): We prove by applying induction on the length of a proof of \$ from M. Base Case: n=1 Then q is either an axiom or, a member of r Then we have that MFQ (see above). Induction flypothesis: (n < m) The repult holds when the length of proof of q from  $\Gamma'$  is  $\leq m$ . Induction Step: n= m+1 Then q is either an aniom, or, a member of F. or obtained by an application of some rule. The first two cases have already been dealt with in the base case. Now, suppose that q is obtained by some rule of the form: Y, , V<sub>2</sub>, -.., Y<sub>K</sub> . Then, Y, , V<sub>2</sub>, --, Y<sub>K</sub> have occurred in the proof of q from T. So, by I.M., we have MEY, MEY2, ..., MEYK. Since, the rule preserves consequence, we

have MFN, that is, MFQ. This completes the proof. Now we have an understanding of what roundness means ! If MFQ then MFQ The converse part (If TFQ then MtQ) is what we call the completeness result. What happens if M= \$ . ? Soundness: If t q then = q. Completiness: If Eq then t q. Note: When I q, we say that q is a theorem In other words, from the definition of MHP, with  $\Gamma = \phi$ , we have the following .  $F \phi$  if there is a sequence of formulas  $Q_1, Q_2, \cdots, Q_n (= Q)$ , s.t. each  $Q_1$  is either a anion or, obtained by the application A some rule.

Classical Propositional Logie. Allphabet {p, 1, 7, V, A, -, 4, (,)}. We will use the following abbreviation! p followed by n 1's is denoted by pn. het P denote a conntable set of these propositional variables pn's Language (d)  $\varphi, \psi := p_n | \neg \varphi | \varphi \lor \psi | \varphi \land \psi | \varphi \rightarrow \psi | \varphi \leftrightarrow \psi$ atomic for mulas Model : , a function from the set  $V: P \rightarrow \{0, 1\}$ variables to {0,1]. We of proposition al a valuation. call this V us Cleim: Any V: P > {0, 13 can be uniquely extended to V: L > {0, 13 .

