Linear Temporal Logic

Sruti Goswami

Introduction

A transition system is a model used to represent behaviour of a system that changes over time. There are 4 components to a transition system. State, actions. transitions and initial state.

Given a finite set of atomic propositions AP, a property P is a set of infinite words over the alphabet, the power set of AP.

A transition system TS satisfies P if the traces of the transition system is contained in P.

Linear temporal logic or LTL is a modal temporal logic used to formalise the properties.

Syntax

A finite set of atomic propositions, logical operators ¬ and ∨, and temporal modal operators X and U.

$\phi := true | p | \phi_1 \vee \phi_2 | \neg\phi | X\phi | \phi_1 U \phi_2$

X is called next, U is called until.

Additionally we have derived logical operators ∧, →, ↔ and derived temporal operators F (in future), G (globally), R (release), W, M.

Semantics

w be a word on a path of a transition system. An LTL formula ϕ satisfies an word w or $w \vDash \phi$ if

1. $w \vDash true$
2. $w \vDash \phi$ if $\phi \in w(0)$
3. $w \vDash \neg\phi$ if $w \nvDash \phi$
4. $w \vDash \varphi \lor \psi$ iff $w \vDash \varphi$ or $w \vDash \psi$
5. $w \vDash X\phi$ if w satisfies ϕ in the next timestep
6. $w \vDash \varphi\ U\ \psi$ if w satisfies ϕ until w satisfies ψ

Similarly we can define for other derived logical operators by the following rules.

Semantics

Derivations of additional logical and temporal symbols

1. $\varphi \land \psi \equiv \neg(\neg\varphi \lor \neg\psi)$
2. $\varphi \rightarrow \psi \equiv \neg\varphi \lor \psi$
3. $\varphi \leftrightarrow \psi \equiv (\varphi \rightarrow \psi) \land (\psi \rightarrow \varphi)$
4. $F\psi \equiv \text{true } U\ \psi$ (eventually ψ becomes true)
5. $G\psi \equiv \neg F \neg\psi$ (ψ is always true)
6. $\psi\ R\ \varphi \equiv \neg(\neg\psi\ U\ \neg\varphi)$

Equivalence of formulas

Two LTL formulas are said to be equivalent if the words satisfying both formulas are the same. Eg

Distributivity:

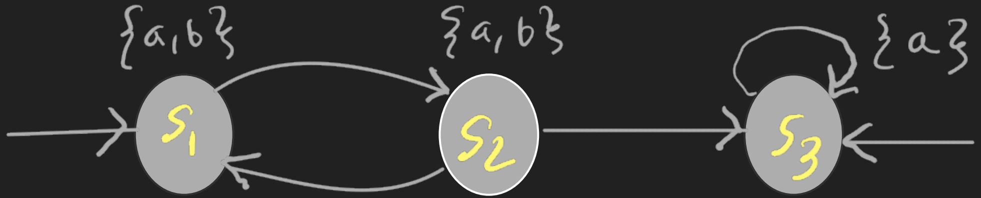
$X (\varphi * \psi) \equiv (X \varphi) * (X \psi)$ where * is $\vee$, $\wedge$, U. $F (\varphi \vee \psi) \equiv (F \varphi) \vee (F \psi)$. $G (\varphi \wedge \psi) \equiv (G \varphi) \wedge (G \psi)$. $\rho \, U (\varphi \vee \psi) \equiv (\rho \, U \varphi) \vee (\rho \, U \psi)$. $(\varphi \wedge \psi) \, U \rho \equiv (\varphi \, U \rho) \wedge (\psi \, U \rho)$

Duality: X is self dual. F and G are dual to each other. $(\neg F \varphi \equiv G \neg\varphi)$

Temporal properties: $*\varphi = **\varphi$ where * is F or G. $\varphi \, U \psi \equiv \varphi \, U (\varphi \, U \psi)$ etc.

Simple example

Consider the following transition system with propositions {a, b}



This transition system satisfies Ga. Also $s_1 \models X(a \land b)$, but $s_2 \not\models X(a \land b)$, $s_3 \not\models X(a \land b)$. So TS $\not\models X(a \land b)$.

Also TS $\models G(\neg b \rightarrow G(a \land \neg b))$

TS |= $bU(a \land \neg b)$.

Safety and Liveness properties using LTL

Safety property:

“Something bad never happens.”

LTL formulation: $G \neg\varphi$

Liveness property:

“Something good keeps happening.”

LTL formulation: $GF\varphi$

LTL based model checking

For a transition system to satisfy a formula (set of infinite word) ϕ, all the traces of the system satisfies ϕ. Alternatively if there exist a trace that satisfies $\neg\phi$, we can say that $TS \nvDash \phi$.

To find that we do the following

1. We convert the LTL formula $\neg\phi$ to a non deterministic Buchi automaton $A_{\neg\phi}$.
2. We convert the TS to a non deterministic Buchi automaton A_{TS}.
3. Check weather $L(A_{\neg\phi}) \cap L(A_{TS})$ is empty. The Buchi automaton corresponding to $L(A_{\neg\phi}) \cap L(A_{TS})$ is $L(A_{\neg\phi} \times A_{TS})$.

References

1. Principles of Model Checking - Joost-Pieter Katoen.
2. Logic in Computer Science - Huth and Ryan.
3. Model Checking course by B. Srivatsan.
4. Wikipedia article on LTL.