FOL EXPRESSIBILITY OF PROPERTIES OF GRAPHS

- Logic for CS Presentation.

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- This is a small disscussion on expressibility of some properties of Graph.

- Not all properties of graph can be expressed in FOL

Eg:- Path, Reachability can't be expressed in FOL.

- Now we are about to see some properties which can be expressed in FOL.

- To do that We need to define

our domain, constants, predicates &
functions of our FOL.

$\mathcal{D} = C = F = \phi$

$\mathcal{P} = \{E, P\}$

Here, E, P are 2 predicate symbols

$E(x, y)$ = denotes the existance of edge in the graph

$P(x, y)$ = denotes the existance of path in the graph.

(i) Simple Graph :-

It is a graph that does not have self loops at any vertex of Graphs.

Axiom :-

$\forall x \; \neg E(x, x)$

(ii) Complete graph :-

In this graph each vertex

has one edge between all other vertices (each vertex is connected to every other vertex)

Axiom:-

$\forall x \forall y \; E(x, y)$

(c) Connectedness:-

We say a graph is connected when there exists a path b/w every pair of vertices.

Axiom:-

$\forall x \forall y \; P(x, y)$

(d) Tree:-

If a graph is connected and acyclic then we say that graph is a Tree

- Tree can be expressed in FOL only if we allow infinite axioms.

if not we can only prove the existance of it by using compactness Theorem.

Axioms:-

(i) $\forall x \forall y \quad P(x,y)$ (connectedness)

(ii) $\forall x_1, \forall x_2, \forall x_3 \quad E(x_1,x_2) \wedge E(x_2,x_3) \rightarrow \neg E(x_3,x_1)$

(No 3-cycle's)

(ii) $\forall x_1, \dots, \forall x_4 \quad E(x_1,x_2) \wedge E(x_2,x_3) \wedge \dots \wedge E(x_3,x_4) \rightarrow \neg E(x_4,x_1)$

(No "4"-cycles)

⋮

$\forall x_1, \dots \forall x_n \quad E(x_1,x_2) \wedge \dots \wedge E(x_{n-1},x_n) \rightarrow \neg E(x_n,x_1)$

(No "n"-cycles).

- As a Tree is Acyclic this implies $n \rightarrow$ "∞"
- Any graph which is a tree should hold these axioms

(e) Bipartite graph:

A graph which has no odd cyclics

. even bipartite graph can only be expressed if we allow infinite axioms. if not we can only prove it's existance by using compactness theorem

Axioms:

(i) $\forall x_1 \forall x_2 \forall x_3 \; E(x_1, x_2) \wedge E(x_2, x_3) \rightarrow \neg E(x_3, x_1)$

(No "3"-cyclics)

(ii) $\forall x_1 \forall x_2, \ldots \forall x_5 \; E(x_1, x_2) \wedge E(x_2, x_3) \wedge \ldots \wedge E(x_4, x_5) \rightarrow \neg E(x_5, x_1)$

(No "5"-cyclics)

$\vdots$

$\forall x_1, \ldots \forall x_{2n+1} \; E(x_1, x_2) \wedge \ldots \wedge E(x_{2n}, x_{2n+1}) \rightarrow \neg E(x_{2n+1}, x_1)$

(No "2n+1" cyclics)

- As there are no-odd cycles in bipartite graphs implies $n \rightarrow \infty$

- Any bipartite graph should hold the above axioms.