A2. Philosophical Logic

Higher-Order Belief Change in a Branching-Time Setting

Virginie Fiutek, Sujata Ghosh and Sonja Smets
University of Groningen, The Netherlands
{V.Fiutek,Sujata.Ghosh,S.J.L.Smets}@rug.nl

Abstract

While the standard work in logic for belief revision after [1] is cast in a syntactic-axiomatic and single-agent setting, recent developments in modal logic show how a semantic approach can give more insight in complicated belief revision scenarios that arise in a multi-agent setting. We consider here two different modal logics for belief change. We start with the branching-time temporal logic developed in [4] and extend this setting with ideas that arise from recent work in Dynamic Epistemic Logic (DEL) [2, 3]. We motivate the extension of Bonanno's logic by considering multi-agent scenarios in which higher-order belief revision plays an important role.

Introduction. In [5], G. Bonanno investigates the interaction of belief and information in the study of belief revision. For his investigation, he uses a temporal multimodal propositional logic consisting of five unary modal operators: the temporal operators F (next instant) and P (previous instant), a belief operator B, an information operator I (restricted to Boolean formulas) and the uniform modality A. While this approach allows one to model the AGM-rules of belief revision in a branching-time setting, it has mainly been pursued in a single-agent context. When more agents are in play, it becomes important to model not only how agents revise their individual beliefs but also their higher-order beliefs (i.e. beliefs about others' beliefs). One also needs to take into account that the information which triggers the agents' to revise their beliefs might not be shared by all agents in the group. This complicates the original setting of Bonanno in two important ways: 1) the incoming information should no longer be restricted to Boolean formulas and 2) the incoming information might not be available to all agents. To see what is at stake, we look at another modal logic in which models have been developed for higher-order belief revision scenarios in a multi-agent setting. We use ideas coming from DEL for belief revision [2, 3] to gain insight in how one can adapt the branching-time setting of Bonanno.

Setting of the Problem. In DEL there is a clear distinction between static and dynamic belief revision. While static belief revision loses track of how the world itself is changed when an agent is confronted with new information that triggers a belief change, dynamic belief revision keeps track of all these changes (see [2]). A typical example is given by the Moore sentence $\varphi = p \wedge \neg Bp$. An agent who is confronted with this information $I(\varphi)$, cannot accept φ afterwards (but might accept that φ was the case before the revision). This indicates that new incoming information can be incompatible with posterior beliefs and highlights a problem that one encounters when holding on to the (static) AGM revision axioms (such as the success axiom). One way out is to follow Zvesper's proposal in [7] of eliminating the Boolean restriction on I-formulas in [4]. In Zvesper's interpretation of the temporal belief revision models, if an agent receives a piece of information at instant t, he revises his beliefs at instant t' where t' is an immediate successor of t – *the information received describes the state of the world as it was before the receipt of this information*. In this paper we agree with Zvesper's solution and will develop this idea further. We also consider scenario's in which the incoming information is not necessarily true, it might come from an untrustworthy or even a highly-trusted but imperfect information-channel and in addition it might not be available to all agents. While an extension of Bonanno's framework in order to deal with public announcements was sketched in [7], it did not yet capture the case when the incoming information might contradict an agent's previous beliefs and neither the case in which the information can be private and possibly false. In order to deal with 'real' belief revision in a multi-agent setting, we have to consider surprising, private and possibly false incoming information. The main goal of this paper is to extend Bonanno's setting in order to handle these more complicated cases. Below we point out how Zvesper's setting can be changed in order to capture real belief dynamics in a temporal-branching framework.

Framework. A temporal belief revision model is a tuple $\langle T, \rightsquigarrow, W, \{B_t, I_t\}_{t\in T}, V\rangle$. The temporal relation, represented by a binary relation $\rightsquigarrow$ on a set of instants T, determines the immediate successors of a given instant. Each instant has a unique predecessor and cycles are excluded. Two binary relations B_t and I_t on a set of states W represent belief and information at each instant t.The valuation function $V : W \to 2^{\mathcal{P}}$ assigns to each state a set of atomic propositions. Zvesper starts from this setting and changes the logic of Bonanno by adding among others the following axioms:

Uniform Announcement (UA) : $I\varphi \to AI\varphi$ Perfect Recall (PR) : $I\varphi \to (B(\varphi \to \psi) \to FBP\psi)$

No Miracles (NM) : $I\varphi \to (\neg F\neg B\neg P\neg\psi \to B(\varphi \to \psi))$

The corresponding semantic properties are:

(1) $I_t(w) = I_t(w')$

(2) if there exists an instant t_0 such that $t_0 \rightsquigarrow t_1$, then $B_{t_1}(w) = I_{t_0}(w) \cap B_{t_0}(w)$

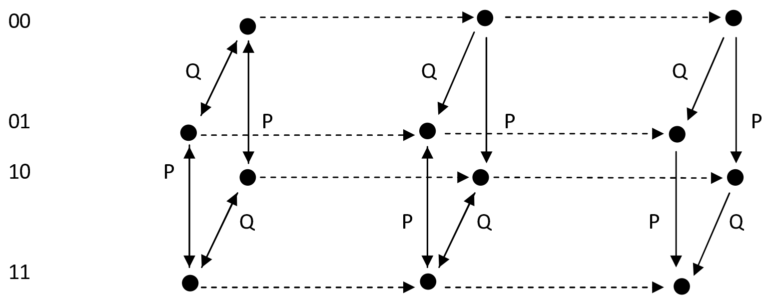
To handle the scenario's we have in mind, we remove the UA axiom in case we consider the information to be private. Hence in our setting different pieces of information can be received in different worlds in the model. Meanwhile we keep the axiom NM and PR but we slightly change the latter. We change PR to $I\varphi \to ((B(\varphi \to \psi) \wedge \neg B\neg\varphi) \to FBP\psi)$ because its original formulation is only valid if the incoming information does not contradict the initial beliefs. In the definition of the model,we only keep property (2) with a slight modification: if there exists an instant t_0 such that $t_0 \rightsquigarrow t_1$, then

if $I_{t_0}(w) \cap B_{t_0}(w) \neq \emptyset$ then $B_{t_1}(w) = I_{t_0}(w) \cap B_{t_0}(w)$

if $I_{t_0}(w) \cap B_{t_0}(w) = \emptyset$ then $B_{t_1}(w) = I_{t_0}(w)$.

The last condition captures what happens when the information contradicts the initial beliefs.

Example. Consider the muddy children puzzle with a different story-line. Three mischievous kids are playing in a garden. One of them suddenly laughs out at the other two (Pat and Que), saying that, at least one of them has mud on her forehead. There is no reason to assume that she is making a 'hard' statement of fact. So in modeling this situation we have to consider all possibilities, e.g. none or one (both) of them has (have) mud on the forehead(s), even after the announcement. The beliefs of the children are represented by two belief modalities : B_p for Pat' s belief and B_q for Que's belief. Two propositional variables *p* and *q* respectively correspond to "Pat is muddy" and "Que is muddy". The model has four states: neither is muddy (00), Pat is muddy (10), Que is muddy (01) and both are muddy (11). The states that the children consider plausible are linked by the corresponding accessibility relations. The figure represents the model structure at three time instants. At time t_0 (model is the left-most structure) the children learn that at least one of them is muddy that is, $I(p \vee q)$ (updated model is the middle structure). At t_1 they learn that neither of them believes that they are muddy that is, $I(\neg B_p p \wedge \neg B_q q)$ (updated model is the right-most structure).



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