Action-information interplay in the cops and robber game (extended abstract)

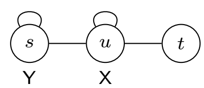
Dazhu Li[1,2], Sujata Ghosh[3] and Fenrong Liu[4]

[1]Institute of Philosophy, Chinese Academy of Sciences
[2]Department of Philosophy, University of Chinese Academy of Sciences, Beijing, China
[3]Computer Science Unit, Indian Statistical Institute, Chennai, India
[4]Joint Research Center for Logic, Department of Philosophy, Tsinghua University, Beijing, China
fenrong@tsinghua.edu.cn

Search missions and pursuit-evasion environments have been widely investigated in the study of multi-agent systems. Such problems can be modeled by pursuit-evasion games or their many-agent counterpart, *cops and robber games* [7]. While prior work has predominantly explored these games from algorithmic and combinatorial perspectives [6, 1], our present research takes a distinctive approach, centering on a logical viewpoint. Specifically, we delve into the players' reasoning abilities, i.e., how they reason about the positions of themselves and their opponents by making use of knowledge and information that they receive during the play, depending on the extent of their observational powers. Our primary focus lies in the dynamic interaction between players, and we introduce a formal framework designed to facilitate the study and analysis of these interactions.

The cops and robber game is played on graphs, and our research is in the tradition of the development of logics describing graph-based games. In this line of research, a pioneer work is that for *sabotage game* [9], which was shown to be a rich platform for studying computational issues in dynamic networks [3]. So far, many graph games have been explored with a modal perspective, including *travel games*, *poison games*, and *hide and seek games* [10]. More specifically, Li et al. [4] introduced a modal language featuring an identity constant I to describe the seeker catching the hider, which was further studied in [8, 2, 5]. But unlike this series of developments that take the modeler's perspective, our work will focus on players' perspectives bringing in uncertainties into the picture. Naturally, a novel epistemic dimension is added to the existing frameworks describing such games. To provide a glimpse of what we will explore, consider the following example within the context of the game:

Example 1. *In the graph below, a cop* X *is at* u *and a robber* Y *at* s*. They know the graph structure and their own positions. Also, we assume that* Y *has the ability to observe a player who is at nodes reachable from her position via an edge, while* X *does not have the ability. Thus,* Y *knows where* X *is, whereas,* X *does not know where* Y *is, but knows that* Y *is either at* s *or* t *(as* X *knows the graph structure).*



A player whose turn it is has to move along an edge. Let the game begin and X *act first. Suppose* X *stays at* u*. Next, when it is* Y*'s turn, between two possible actions: moving to* u *and staying at* s*, she naturally chooses the latter, otherwise she would be caught at* u*. However, since* X *knows the graph,* Y*'s action makes* X *realize that* Y *cannot be at* t*, as otherwise,* Y *would have been at* u *where* X *is, after* Y*'s turn. Based on the fact that* X *is not at the same position with* Y*,* X *knows that* Y *is at* s *and wins.*

Looking at the example above, many natural questions arise: What is meant by 'knowing a graph'? What is meant by 'players having observational powers'? How meaningful are all these assumptions for the cops and robber game? And finally, is there a formal tool to reason about the knowledge of the players during the play? To make these clear, in the article we introduce a formal device called *Logic of the Cops and Robber Game* LCR. Its language $\mathcal{L}$ is based on a vocabulary $Voc = (Pred, Var)$, where $Pred$ is a set of predicate symbols, containing a specific binary relation R describing the edges of a game graph, and Var is a non-empty, finite set of variables, representing the players. Precisely,

$$\mathcal{L} \ni \varphi ::= P\boldsymbol{x} \mid x \equiv y \mid K_x y \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_x\varphi \mid [x]\varphi$$

where $x, y \in Var$, $P \in Pred$, and $\boldsymbol{x}$ is a tuple of variables. The interpretations for $P\boldsymbol{x}$, $x \equiv y$ and Boolean connectives $\neg$, $\wedge$ are the same as that in first-order logic, $K_x y$ reads *the player* x *knows the position of the player* y, $K_x\varphi$ means x *knows that* φ, and $[x]\varphi$ states φ *is the case after any movement of* x. Clearly, the meanings of the formulas need to be defined in a precise way, for which it is crucial to develop an appropriate semantic proposal for operators $[x]\varphi$: they do not only change positions of players, but also cause the knowledge of all players to change (based on their existing knowledge and observational powers). In the work we will present a desired semantic approach, with which we can show the following:

- The assumptions about the players, including their knowledge about graph structures and their observational powers, can be characterized with LCR. Moreover, LCR can also define the winning positions for the players and has desired applications to the games.

- The model-checking problem for the fragment of LCR without operators $[x]$ is P-complete.
- The satisfiability problem for the fragment of LCR without operators $[x]$ is decidable.
- In addition, we also consider the meta-properties for the whole LCR, including its complexity and axiomatization, for which we develop different, yet equivalent, semantic proposals for the logic to facilitate utilizing techniques from different fields.

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