A modal logic for reasoning in distributed games

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Abstract

In distributed games, every player sees only the local game arena, and announces potential joint moves with other players. The global arena resolves these and the game proceeds. We propose a two-level logic to reason in such games, with one layer of local formulas for each player, and the global formulas. We present a complete axiom system for valid formulas and prove decidability.

Talk Proposal

Distributed systems have been widely studied in computer science and AI applications. According to [3], a distributed system is a collection of autonomous computing elements that appears to its users as a single coherent system. These computing elements are independent of each other but can communicate or share messages depending on various patterns. In this work, a distributed system is seen as a concurrent system of parallel processes, represented by labeled transition models.

[1] brought a game theoretical perspective to distributed systems. According to this perspective, we treat a component process as a local arena for one player, while the distributed system consisting of all these processes is the global arena. A similar formulation on local and global arena was provided by [2]. In these games, a player makes choices locally, based on her position in the local arena and information received from other players by public announcements, which allows the player to make assumptions about other player's local game states. The product arena resolves conflicts and ambiguities arising from such partial knowledge. A simple card game described below, demonstrates this viewpoint.

Example 1. *Alice and Bob play a card game with five cards this time, say, 1,2,3,4 and 5. They are dealt two cards each, and we suppose that Alice gets cards 1 and 4, and Bob gets cards 2 and 3, and they can only see their own cards. The rules of the game are as follows:*

- *Alice and Bob play in turns.*
- *They can choose to throw one card or jump (Q).*
- *If one player throws a card, then the other player has to throw a bigger card or jump.*
- *If one player chooses to jump, then the other player can only choose to throw a card.*

The winning condition *is given as follows:*

- *The player holding no card first wins the game.*

Note that 'throwing a card' can be identified with 'making an announcement about a card' in terms of abstract moves. The game starts with one of the players, Alice say, throwing a card. We present the corresponding local arenas in figure 1. We note that these figures do not show all the moves explicitly in the players' local arenas, they are basically used to illustrate certain instances of the game we are talking about.

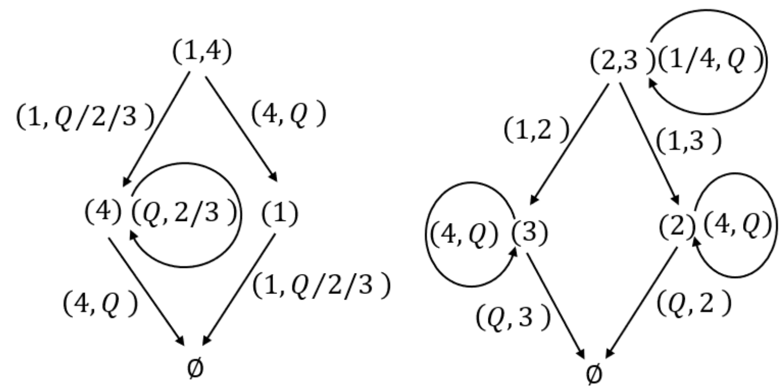

Figure 1: A local arena for Alice (left) and a local arena for Bob (right)

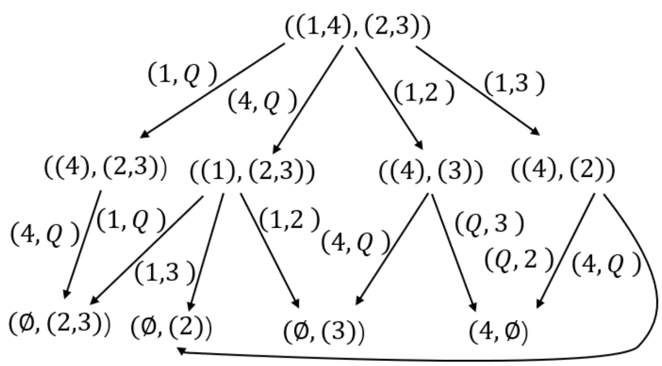

Figure 2: A global arena

We reiterate that the local arenas are game processes from the perspective of players playing the game, while the global arena is the game process from the perspective of those watching the game from top, that is, those who can reason about *the game. They can clearly identify the enabled moves at each game state and the outcomes of the game. In the following, we explain the symbols and notations used in figure 1:*

- *Each node in a local arena for player i is a state depicting the cards that player i holds at a certain stage of the game.*
- *The symbol $\emptyset$ is used to denote that player i holds no card.*
- *The action symbol Q is used to denote that player i jumps.*

- *Each move in the local arena represents a pair of actual moves in the game as follows: (Alice's move, Bob's subsequent move).*

At the beginning of the game described above, Alice holds cards 1 *and* 4*, while Bob holds cards* 2 *and* 3*. Suppose Alice plays card* 4*, Bob then chooses to jump, after that Alice only holds card* 1 *and Bob still holds cards* 2 *and* 3*. From the perspective of Alice, there is a game state transition from* $(1, 4)$ *to* (1) *through the move* $(4, Q)$*, while for Bob, there is a state transition from* $(2, 3)$ *to* $(2, 3)$ *through the* $(4, Q)$*. Correspondingly in Figure 2, from the perspective of the modeler, there is a state transition from* $((1, 4), (2, 3))$ *to* $((1), (2, 3))$ *through the* $(4, Q)$ *in a round.*

In the above example, we have shown how to model a card game using local and global arenas. Based on the local and global arenas, we will propose local and global models, we won't give detailed definitions here. Next, we will propose a modal logic, viz. Distributed Game Logic (DGL), to reason about such a combination of local and global reasoning in local and global models. The formulas of the logic are presented in two layers: *local formulas* describing the reasoning of individual players and *global formulas* describing the reasoning of the game depending on the initial states. We present the language as follows.

Definition 1. *Let* P_i *be a set of atomic propositions for each player* i*, the local language for player* i*,* $\mathcal{L}_i$*, is given as follows:*

$$\alpha \in \mathcal{L}_i ::= p \mid \neg\alpha \mid \alpha \vee \alpha' \mid \langle\gamma\rangle\ \alpha,$$

where, $p \in \mathsf{P}_i$ *and* $\gamma \in \tilde{\Gamma}$*.*

Definition 2. *The global language* $\mathcal{L}$ *is given as follows:*

$$\varphi \in \mathcal{L} ::= \alpha@i, \alpha \in \mathcal{L}_i \mid \neg\varphi \mid \varphi \vee \varphi' \mid \langle\gamma\rangle\ \varphi$$

As in the case of local formulas, the global formulas are also quite simple as basic modal logic, the only difference being that we use annotated local formulas as atomic global formulas.

Next, we will propose a complete axiomatization of validities. To prove the completeness, i.e. Every consistent set of formulas Φ is satisfiable, we can extend Φ to a global MCS A and specify the induced i-local MCS $(A)_i$ as the initial state in i-local model for $i \in N$, then construct the global model with all local models, and we have to show the feasibility from a syntactic perspective, which involves proving the reduction theorem, i.e., every global formula φ has an equivalent formula without the modality $\langle\gamma\rangle$, then we can establish the *truth lemma* at the global level (by utilizing the *truth lemma* at the local level).

Moreover, we will show the logic DGL is decidable. Fix a global formula φ_0 and consider the set of its subformulas, closed under negation, denoted $SF(\varphi_0)$. For each i, we can then define $SF_i(\varphi_0)$ such that this is also closed under subformulas and negation, and further, $\alpha \in SF_i(\varphi_0)$ iff $\alpha@i \in SF(\varphi_0)$.

We also need to define global atoms, (good) global atom graphs, i-local atoms and (good) local atom graphs. Then we give a main lemma asserts that a formula φ_0 is satisfiable iff G_{φ_0} contains a good subgraph. According to the lemma, we can obtain a decision procedure to test whether φ_0 is satisfiable.

At the end of this talk, we will conclude our talk by engaging in further discussion on strategic reasoning. This will involve expanding and modifying our previous framework.

References

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