

Thesis

Mathematical modelling and analysis of reliability in the context of product warranty service

Abstract: The thesis is comprised of six chapters. A brief account of the work presented in the thesis is given below.

In the first chapter, some basic concepts on product reliability and warranty, and some modelling and decision problems related to product warranty service are described. Importance of modeling and analysis of reliability in the present context is highlighted.

Chapter 2 deals with failure models for warranty. Importance of failure modelling under warranty, especially modelling of first failure instance, is highlighted. The existing first failure models under 2D warranty are enumerated and their suitability in the context of component failure modelling are discussed. A new methodology for the derivation of a probability model using a use-rate based approach applicable for a component under 2D warranty is presented. The key feature of this model is: higher product use-rate hastens component failure. Numerical examples are provided for the computation of expected number of failures during the warranty period illustrating the application of the proposed model. [Part of research work of this chapter is published as: A use-rate based failure model for two-dimensional warranty, *Computers & Industrial Engineering*, 52, 229-240, 2007.]

In chapter 3, the effect of use-rate on the first failure instance of a system is investigated. Specifically, it is verified whether higher use-rate hastens first failure of the system too. Failure models are derived and analyzed with an assumption that components of a system are conditionally independent. The effect of use-rate on failure age of system with a general joint distribution of component failure times is also investigated. [Research work of this chapter is published as: Effect of Use-rate on System Life and Failure Models for 2D Warranty. *International Journal of Quality and Reliability Management*, 28(4), 464 - 482, 2011.]

In chapter 4, we consider the problem of calculating the expected number of failures in a rectangular two-dimensional warranty region for both types of products: non-repairable, and repairable with minimal repair. Specifically, we demonstrate through examples the discrepancy in the formulae for the expected number of failures derived by the two approaches, namely, one-dimensional and two-dimensional. In particular, we raise some issues against possible wrong use or interpretation of the formulae. We also propose a correction in the formula and discuss some issues concerned with repairable product. [Research work of this chapter is published as: A note on calculating cost of two-dimensional warranty policy. *Computers & Industrial Engineering*, 54(4), 1071-1077, 2008.]

In chapter 5, we shift our focus to the determination of optimal warranty region for a two-dimensional policy. We attempt here to find the optimal region that suits the customers best, in some sense, when manufacturer specifies the budget constraint on warranty cost. The proposed methodology discusses derivation of optimal region among fixed and flexible policies. This is illustrated through a set of assumptions, namely, Lognormal usage function, and Weibull failure process under minimal repair. With these assumptions, we show that flexible rectangular warranty region is the best. [Research work of this chapter is published as: Optimal determination of warranty region for 2D policy: A customers' perspective. Computers & Industrial Engineering, 50, pp.161-174, 2006.]

In chapter 6, we present three real-life cases on warranty cost estimation. Each case involves modelling and estimation of warranty cost from the field failure data, and prediction of the expected warranty cost per unit when the existing warranty period is extended.

The first case study, described in section 6.2, deals with warranty cost estimation of Temperature Switch-cum-Sensor (TSS) that is offered with a one-dimensional renewing free-replacement warranty policy. Life distribution of TSS is identified as Weibull from warranty failure data. Then warranty cost per unit and its confidence interval estimates are computed for the extended time period using bootstrap method. [This case study is published as: Cost estimation under renewing warranty policy – an application. Quality Engineering, 16(1), pp.95-100, 2003.]

In the second case study, described in section 6.3, an application of Gumbel's bivariate exponential distribution model in the context of estimating warranty costs of motor cycles under a two-dimensional FRW policy is presented. The problem is solved by studying the underlying renewal process. Some practical difficulties posed by the past data in the analysis are highlighted and tackled in an interesting way. [This case study is published as: An application of Gumbel's bivariate exponential distribution in estimation of warranty cost of motor cycles. International Journal of Quality & Reliability Management, 20(4), pp.488-502, 2003.]

The third case study, described in section 6.4, deals with the problem of cost estimation for extended warranty period of a multi-module product under a 2D FRW policy. From the data, it is observed that age and usage are highly correlated. Based on life (age) data, the joint life distribution of the modules is well-described by Multivariate Exponential distribution of Marshall and Olkin. The same is utilized to estimate cost for desired warranty times by the method of simulation. [This case study is published as: Warranty cost estimation of a multi-module product. International Journal of Quality & Reliability Management, 21(1), pp.102-117, 2004.]
